ESTD. 2001

LEARN TO LEAD

STEPPING STONE SCHOOL (HIGH)

CLASS: IX

SUBJECT - MATH

TOPIC : MENSURATION

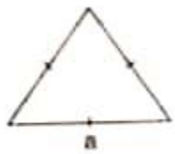
Dated : 26.06.2020

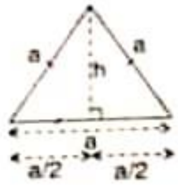
WORKSHEET # 23

SOME SPECIAL TYPES OF TRIANGLES

1. Equilateral Triangle :

Let the length of each side of an equilateral triangle be a unit; then, its perimeter $= 3 \times$ its side $= 3a$





$$\text{and its area} = \frac{\sqrt{3}}{4} \times (\text{side})^2 = \frac{\sqrt{3}}{4} \cdot a^2$$

> **2** The area of an equilateral triangle is numerically equal to its perimeter. Find a side of the triangle [Take $\sqrt{3} = 1{\cdot}73$].

Solution :

Given : Area = Perimeter [Numerically]

$\Rightarrow \quad \frac{\sqrt{3}}{4}(\text{side})^2 = 3 \text{ side} \quad i.e. \quad \text{side} = \frac{3 \times 4}{\sqrt{3}}$

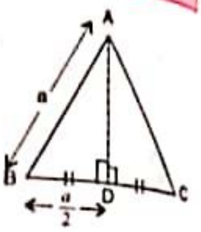
$\Rightarrow \quad$ **side** $= 4\sqrt{3} = 4 \times 1{\cdot}73$ unit $=$ **6·92 unit**

Ans.

> **3** Calculate the area of an equilateral triangle, whose height is 20 cm.

Solution :

Let ABC be the given equilateral triangle and AD is perpendicular to base BC; then clearly; AD = 20 cm



If each side of the given triangle be a cm; then AB = a cm

and, $BD = \frac{1}{2} BC$ [In equilateral Δ, perpendicular from vertex bisects the base]

$= \frac{1}{2} a$ cm

In right-angled triangle ABD :

$$AD^2 + BD^2 = AB^2 \quad \Rightarrow (20)^2 + \left(\frac{a}{2}\right)^2 = a^2$$ [Pythagoras Theorem]

On simplifying, we get : $a^2 = 400 \times \frac{4}{3} = \frac{1600}{3}$

$\therefore$ **Area of the triangle** $= \frac{\sqrt{3}}{4} a^2$

$= \frac{\sqrt{3}}{4} \times \frac{1600}{3}$ cm^2 = **230·9 cm^2** Ans.

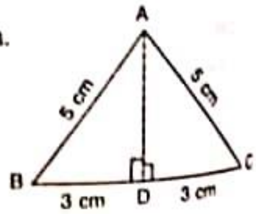
2. Isosceles Triangle :

> **4** Find the area of an isosceles triangle whose equal sides are 5 cm each and base is 6 cm.

Solution :

In an isosceles triangle ABC, let AB = AC = 5 cm and BC = 6 cm.

Draw AD perpendicular to BC.



Since, the perpendicular from the vertex to the base of an isosceles triangle bisects the base, therefore

$BD = CD = \frac{1}{2} \times 6 \text{ cm} = 3 \text{ cm}$

Applying Pythagoras Theorem in triangle ABD, we get :

$AD^2 = AB^2 - BD^2$

$= 5^2 - 3^2 = 25 - 9 = 16 \quad \Rightarrow AD = 4$ cm

$\therefore$ Area of $\Delta = \frac{1}{2}$ base × height

$= \frac{1}{2} BC \times AD = \frac{1}{2} \times 6 \times 4 \text{ cm}^2 = 12 \text{ cm}^2$ **Ans.**

Alternative method :

Since, the sides of the given isosceles triangle are 5 cm, 5 cm and 6 cm

$\therefore s = \frac{5+5+6}{2}$ cm $= 8$ cm

and **area of Δ** $= \sqrt{8(8-5)(8-5)(8-6)}$ cm²

$= \sqrt{8 \times 3 \times 3 \times 2}$ cm² $=$ **12 cm²** **Ans.**

Third method :

Area of an isosceles triangle

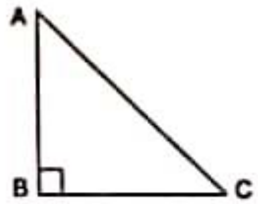
$= \frac{1}{4} \times b \times \sqrt{4a^2 - b^2}$; where, a = length of each equal side

and, b = length of base.

$= \frac{1}{4} \times 6 \times \sqrt{4 \times 5^2 - 6^2} = 12 \text{ cm}^2$ **Ans.**

3. **Right-angled Triangle :**

The area of a right-angled triangle is equal to half the product of the sides containing the right angle.



In the given figure; Area of Δ ABC $= \frac{1}{2}$ AB × BC

5 **The sides of a triangle containing the right angle are 5x cm and (3x – 1) cm. If the area of the triangle is 60 cm², calculate the lengths of the sides of the triangle.**

Solution :

Since, area of a right-angled triangle $= \frac{1}{2}$ × product of its sides containing the right angle

$\therefore \quad 60 = \frac{1}{2} \times 5x \times (3x - 1)$

$\Rightarrow \quad 120 = 15x^2 - 5x$ *i.e.*, $3x^2 - x - 24 = 0$ [Dividing each term by 5]

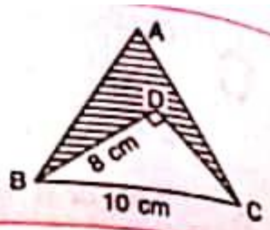
On solving the quadratic equation; we get : $x = 3$ and $x = \frac{-8}{3}$

Since, $x = \frac{-8}{3}$ will give negative values of the sides of the triangle, which is impossible; therefore, $x = 3$.

$\therefore$ **Lengths of the sides** $= 5x$ cm and $(3x - 1)$ cm

$= 5 \times 3$ cm and $(3 \times 3 - 1)$ cm $=$ **15 cm and 8 cm** **Ans.**

6 The given figure shows an equilateral triangle ABC whose each side is 10 cm and a right-angled triangle BDC whose side BD = 8 cm and $\angle D = 90°$. Find the area of the shaded portion.



Solution :

In right-angled Δ BDC,

$$BD^2 + CD^2 = BC^2 \Rightarrow 8^2 + CD^2 = 10^2$$
$$\Rightarrow CD = 6 \text{ cm}$$

$\therefore$ Area of Δ BDC $= \frac{1}{2} \times BD \times CD = \frac{1}{2} \times 8 \text{ cm} \times 6 \text{ cm} = 24 \text{ cm}^2$

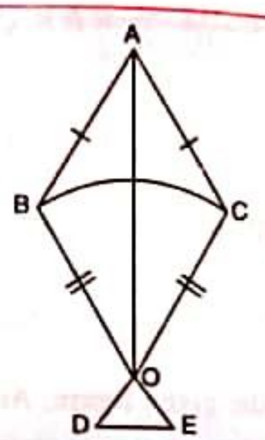
Also, area of equilateral Δ ABC $= \frac{\sqrt{3}}{4} \times (\text{side})^2 = \frac{1.732}{4} \times 10^2 \text{ cm}^2 = 43.3 \text{ cm}^2$

$\therefore$ The area of the shaded portion

$= $ Area of Δ ABC $-$ Area of Δ BDC

$= 43.3 \text{ cm}^2 - 24 \text{ cm}^2 = 19.3 \text{ cm}^2$ **Ans.**

7 A kite is made as shown alongside in which ABC is an equilateral triangle with side 20 cm, BOC is an isosceles triangle with OB = OC = 26 cm and ODE is an isosceles triangle with base DE = 8 cm and height 6 cm. Find the whole area of the kite.



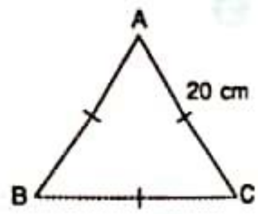
Solution :

Area of $\Delta ABC = \frac{\sqrt{3}}{4} \times (20)^2$ sq. cm

$= \frac{\sqrt{3}}{4} \times 20 \times 20$ sq. cm

$= 100 \times 1.732$ sq cm

$= 173.2$ sq. cm



Join BC and draw OP $\perp$ BC.

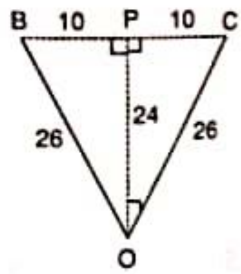
Since, BOC is an isosceles triangle, OP will bisect BC

$\Rightarrow$ BP = CP = 10 cm

In right angle Δ OBP,

BP = 10 cm and OB = 26 cm

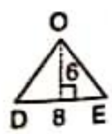
$\Rightarrow$ OP = 24 cm [Using Pythagoras Theorem]



$\therefore$ Area of Δ BOC $= \frac{1}{2} \times BC \times OP$

$= \frac{1}{2} \times 20 \times 24$ sq cm $= 240$ sq cm

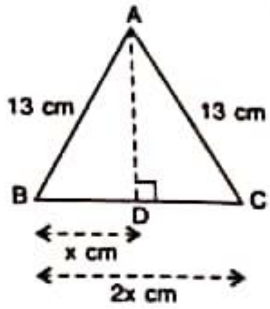
Area of Δ ODE $= \frac{1}{2} \times 8 \times 6$ sq cm $= 24$ sq cm



$\therefore$ Required area $= (173.2 + 240 + 24)$ sq cm $= 437.2$ sq cm. **Ans.**

8 If the area of an isosceles triangle is 60 cm^2 and the length of each of its equal sides is 13 cm, find its base.

Solution :



Let base = $2x$ cm *i.e.* BC = $2x$ cm

$\because$ In an isosceles triangle, the perpendicular from the vertex bisects the base

$\Rightarrow$ BD = CD = x cm

In right-triangle ABD,

$$AD^2 + BD^2 = AB^2$$

$\Rightarrow$ $AD^2 + x^2 = 13^2$

i.e. $AD^2 = 169 - x^2$ and $AD = \sqrt{169 - x^2}$ cm

Given, area of the triangle = 60 cm^2

$\Rightarrow$ $\frac{1}{2}BC \times AD = 60$ *i.e.* $\frac{1}{2} \times 2x \times \sqrt{169 - x^2} = 60$

$\Rightarrow$ $x\sqrt{169 - x^2} = 60$ *i.e.* $x^2(169 - x^2) = 3600$

$\Rightarrow$ $x^4 - 169x^2 + 3600 = 0$ *i.e.* $x^4 - 144x^2 - 25x^2 + 3600 = 0$

$\Rightarrow$ $x^2(x^2 - 144) - 25(x^2 - 144) = 0$ *i.e.* $(x^2 - 144)(x^2 - 25) = 0$

$\Rightarrow$ $x^2 - 144 = 0$ or $x^2 - 25 = 0$ *i.e.* $x = 12$ or $x = 5$

$x = 12$ $\Rightarrow$ base = $2x$ cm = 2×12 cm = **24 cm** **Ans.**

$x = 5$ $\Rightarrow$ base = $2x$ cm = 2×5 cm = **10 cm** **Ans.**

EXERCISE

1. Find the area of a triangle whose sides are 18 cm, 24 cm and 30 cm.

   Also, find the length of altitude corresponding to the largest side of the triangle.

2. The lengths of the sides of a triangle are in the ratio 3 : 4 : 5. Find the area of the triangle if its perimeter is 144 cm.

3. ABC is a triangle in which AB = AC = 4 cm and $\angle A = 90°$. Calculate :

   (i) the area of Δ ABC,

   (ii) the length of perpendicular from A to BC.

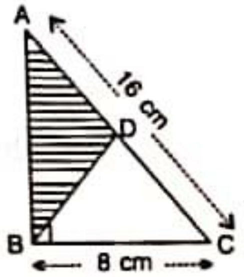
4. The area of an equilateral triangle is $36\sqrt{3}$ sq. cm. Find its perimeter.

5. Find the area of an isosceles triangle with perimeter 36 cm and base 16 cm.

6. The base of an isosceles triangle is 24 cm and its area is 192 sq. cm. Find its perimeter.

7. The given figure shows a right-angled triangle ABC and an equilateral triangle BCD.

   Find the area of the shaded portion.



8. Find the area and the perimeter of quadrilateral ABCD, given below; if, AB = 8 cm, AD = 10 cm, BD = 12 cm, DC = 13 cm and $\angle DBC = 90°$.