ESTD. 2001

LEARN TO LEAD

STEPPING STONE SCHOOL (HIGH)

CLASS: IX

SUBJECT - MATH

TOPIC : MENSURATION

Dated : 25.06.2020

WORKSHEET # 22

AREA AND PERIMETER OF PLANE FIGURES

Perimeter: The perimeter of a plane figure is the length of its boundary.

The unit of perimeter is the unit of length.

Area: The area of a plane figure is a measure of the surface enclosed by its boundary. It is measured in square units.

Example: For small regions, standard units of area, sq. cm (cm^2) are used. For larger regions, sq. m (m^2) and sq. km (km^2) are used. Fields are generally measured in hectares.

100 hectares = 1 sq. km,

1 hectare = 10000 m^2

1 are = 100 m^2

Area and Perimeter of Triangles

1. **Area of a triangle =** $\frac{1}{2}$ × base × height

Any side of a triangle can be taken as base, then the corresponding altitude (height) is used.

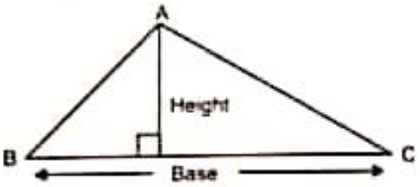


Fig. 17.1

Perimeter of $\Delta ABC = AB + BC + AC$

In obtuse triangle, height is outside triangle as shown in Fig. 17.2(*i*).

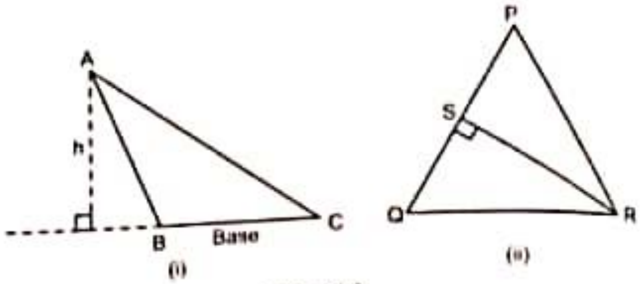


Fig. 17.2

In ΔPQR [Fig. 17.2(*ii*)], if PQ is the base, then RS is the height.

2. **Right-angled Triangle:** In a right-angled triangle, the two sides containing the right angle are the base and height.

$$\text{Area of } \Delta PQR = \frac{1}{2} \times PQ \times QR$$

where $\angle Q = 90°$

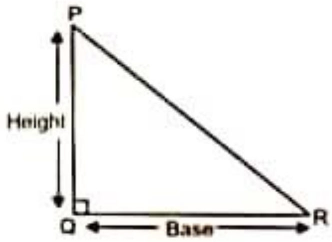


Fig. 17.3

Perimeter of $\Delta PQR = PQ + QR + PR$

3. **Equilateral Triangle:** In an equilateral triangle, the altitude bisects the side.

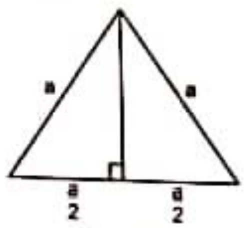


Fig. 17.4

Using Pythagoras theorem,

$$\text{Altitude} = \sqrt{a^2 - \frac{a^2}{4}} = \frac{\sqrt{3}}{2}a$$

$$\text{Area of equilateral } \Delta = \frac{1}{2}a \times \left(\frac{\sqrt{3}}{2}\right)a$$

$$= \frac{\sqrt{3}}{4}a^2$$

Perimeter of equilateral triangle = $3a$

4. **Isosceles Triangle:** In an isosceles triangle also, the altitude on the unequal side bisects it.

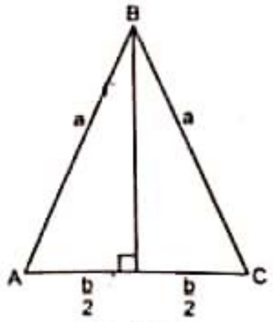


Fig. 17.5

Height can be found using Pythagoras theorem. If the equal sides = a and base = b.

$$\text{Height on the base} = \sqrt{a^2 - \frac{b^2}{4}} = \frac{\sqrt{4a^2 - b^2}}{2}$$

$$\therefore \text{Area} = \left(\frac{1}{2} \times b\right)\left(\frac{\sqrt{4a^2 - b^2}}{2}\right) = \frac{1}{4}b\sqrt{4a^2 - b^2}$$

Perimeter of $\Delta ABC = 2a + b$

Heron's Formula

When all three sides of a triangle are given.

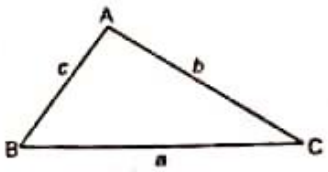


Fig. 17.6

$$\text{Area } \Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{where } s = \text{Semi-perimeter} = \frac{a+b+c}{2}$$

Example 1: Find the area of a triangle with sides 25 cm, 25 cm and 30 cm.

Solution: $s = \dfrac{25 + 25 + 30}{2} = 40$

Using Heron's formula,

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{40(40-25)(40-25)(40-30)}$$

$$= \sqrt{40 \times 15 \times 15 \times 10} = 15 \times 20 = 300 \text{ cm}^2$$

Example 2: Find the area of an equilateral triangle with side 12 cm.

Solution: Area of equilateral triangle

$$= \frac{\sqrt{3}}{4} \times \text{side}^2 = \frac{1.732}{4} \times 12 \times 12$$

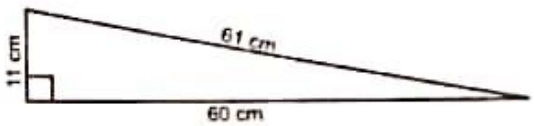
$$= 1.732 \times 36 = 62.352 \text{ cm}^2$$

Example 3: Find the area of a triangle with sides 11 cm, 60 cm and 61 cm.

Solution: Observe that $11^2 + 60^2 = 61^2$

$\therefore$ It satisfies Pythagoras theorem.

Thus, it is a right-angled Δ.

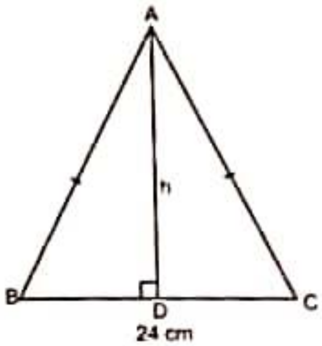


$$\text{Area of } \Delta = \frac{1}{2} \times 11 \times 60 = 330 \text{ cm}^2$$

Note: Recognising a few Pythagoreans triplets helps in finding the area in a simple way.

Example 4: The base of an isosceles Δ is 24 cm and its area is 192 cm^2. Find its perimeter.

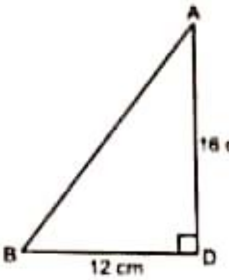
Solution:



Area of $\Delta = \frac{1}{2} \times b \times h = 192$

$\frac{1}{2} \times 24 \times h = 192$

$\Rightarrow \quad h = 16$ cm

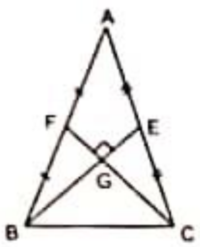


In ΔABD, $h = AD = 16$ cm, $b = BD = 12$ cm

$\therefore \quad AB = \sqrt{12^2 + 16^2} = 20 \text{ cm} = AC$

$\therefore \quad$ Perimeter $= 24 + 20 + 20 = 64$ cm

Example 5: In ΔABC, medians BE and CF intersect at G at right angles. BE = 15 cm, CF = 12 cm. Find the area of ΔABC.



Solution:

Medians intersect at centroid in the ratio 2 : 1.

$CG = \frac{2}{3} \times CF = \frac{2}{3} \times 12 = 8$ cm

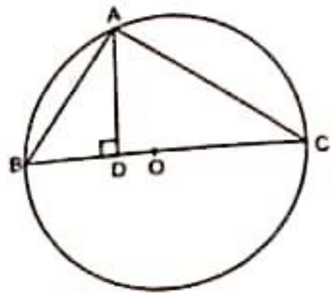
Area of $\Delta BEC = \frac{1}{2} \times BE \times CG$

$= \frac{1}{2} \times 15 \times 8 = 60 \text{ cm}^2$

Area of $\Delta ABC = \frac{}{2} \times$ Area of $\Delta BEC = 120 \text{ cm}^2$

[Median bisects the Δ into 2 equal areas]

Example 6: The radius of the circumcircle of a right-angled triangle is 6 cm and the altitude drawn to the hypotenuse is 4.5 cm. Find the area of the triangle.



Solution:

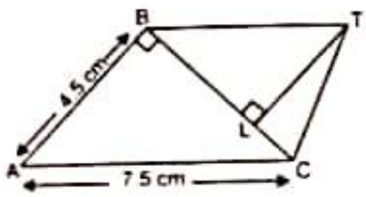
$\therefore \quad$ Hypotenuse $BC = 2r = 12$ cm

Altitude, $AD = 4.5$ cm

Note: In a right-angled Δ, the circumcentre lies at the mid-point of the hypotenuse.

$\therefore \quad$ Area of $\Delta ABC = \frac{1}{2} \times 12 \times 4.5 = 27 \text{ cm}^2$

Example 7: In the figure, ΔABC is right-angled at B. AC = 7.5 cm, AB = 4.5 cm. TL is perpendicular to BC. Calculate TL if the area of the quadrilateral ABTC is 18 cm².



Solution: Using Pythagoras theorem,

$BC = \sqrt{7.5^2 - 4.5^2} = 6$ cm

Area of $\Delta ABC = \frac{1}{2} \times 4.5 \times 6 = 13.5 \text{ cm}^2$

$\therefore \quad$ Area of ΔBTC = Area of quadrilateral ABTC − Area of ΔABC

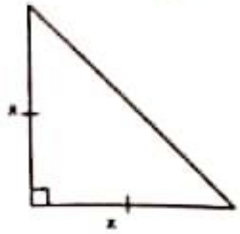
$= 18 - 13.5 = 4.5 \text{ cm}^2$

$\frac{1}{2} \times BC \times TL = 4.5$

$\therefore \quad TL = \frac{2 \times 4.5}{6} = 1.5$ cm

Example 8: The area of an isosceles right-angled triangle is 72 cm². What is the length of its hypotenuse?

Solution: Let x be the length of equal sides.



$$\text{Area of } \Delta = \frac{1}{2}\, x \times x = 72$$

$$x^2 = 144 \Rightarrow x = 12$$

$$\therefore \text{Hypotenuse} = \sqrt{x^2 + x^2}$$

$$= \sqrt{144 + 144} = \sqrt{2 \times 144}$$

$$= 12\sqrt{2} = 12 \times 1.414 = 16.968 \text{ cm}$$

Example 9: The base of a triangular field is double its height. The cost of cultivating the field at ₹360 per hectare is ₹5760. Find its base and height.

Solution: Let the height be x m.

$\therefore$ Base $= 2x$

$$\text{Area of } \Delta = \frac{1}{2} \times 2x \times x = x^2$$

$$\text{Area of the field} = \frac{\text{Total cost}}{\text{Rate per hectare}}$$

$$= \frac{5760}{360} = 16 \text{ hectares}$$

$\therefore x^2 = 160000 \text{ m}^2$ [$\because$ 1 hectare $= 10000 \text{ m}^2$]

$\Rightarrow x = \sqrt{160000} = 400$ m

$\therefore$ Base $= 800$ m and height $= 400$ m

Example 10: The area of a triangle is 48 cm². Find the base if the altitude exceeds the base by 4 cm.

Solution: Let the base be x cm.

$\therefore$ Altitude $= (x + 4)$ cm

$$\text{Area} = \frac{1}{2} \times x(x+4) = 48$$

$$x^2 + 4x = 96$$

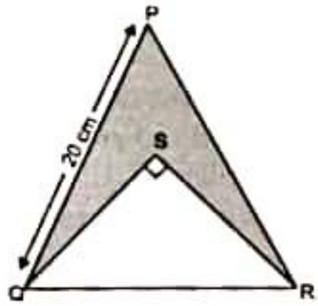
$$x^2 + 4x - 96 = 0$$

$$(x + 12)(x - 8) = 0$$

$\therefore$ $x = 8 \Rightarrow$ Base $= 8$ cm

EXERCISE

1. Find the area of triangle with following sides. [All measures are in cm.]
   (*i*) 10, 17, 21 (*ii*) 17, 25, 28 (*iii*) 25, 39, 40
2. The sides of a triangle are in the ratio 5 : 12 : 13. If its perimeter is 90 cm, find the area of the triangle.
3. In ΔABC, $\angle A = 90°$, $AB = 14$ cm, $AC = 48$ cm. Find the
   (*i*) area of ΔABC
   (*ii*) length of perpendicular from A to BC
4. In ΔPQR, $\angle Q = 90°$ and $PQ = QR = 6$ cm. Calculate the
   (*i*) area of triangle
   (*ii*) length of perpendicular from Q to PR
   [Take $\sqrt{2} = 1.414$]
5. Find the area of an isosceles right triangle with hypotenuse 40 cm.
6. The base of an isosceles triangle is 40 cm and its area is 420 cm². Find the length of its equal sides.
7. In an isosceles triangle, the unequal side is 22 cm and perimeter is 144 cm. Find its area.
8. Find the area of an equilateral triangle with side 8 cm. [Take $\sqrt{3} = 1.732$].
9. If the area of an equilateral triangle is $25\sqrt{3}$ cm², find its perimeter.
10. In the given figure, PQR is an equilateral triangle of side 20 cm. ΔQSR is inscribed in it, $\angle QSR = 90°$, $QS = 16$ cm. Find (*i*) SR, (*ii*) the area of the shaded portion. [Take $\sqrt{3} = 1.732$].



SUGGESTED LINKS

1. https://youtu.be/zLyvu3gUAC8