ESTD. 2001

LEARN TO LEAD

STEPPING STONE SCHOOL (HIGH)

CLASS: IX

SUBJECT - MATH

TOPIC : STATISTICS

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WORKSHEET # 16

20.3 GROUPED OR CLASSIFIED DATA

Consider the following examples of grouped frequency distribution :

Example 1 Using class intervals 1 – 5, 6 – 10, 11 – 15, ... construct the frequency distribution for the following data:

13, 6, 12, 9, 11, 14, 2, 8, 18, 16, 9, 13, 17, 11, 19, 6, 7, 12, 22, 21, 18, 1, 8, 12, 18.

Solution. The frequency distribution table for the given grouped data is:

Class-intervals	*Tally marks*	*Frequency*
1 – 5	II	2
6 – 10	~~IIII~~ II	7
11 – 15	~~IIII~~ III	8
16 – 20	~~IIII~~ I	6
21 – 25	II	2
Total		**25**

Example 2 The following is the pocket money survey of 50 students in a school (pocket money in rupees per month):

49, 55, 22, 27, 30, 27, 25, 27, 30, 42, 40, 13, 24, 38, 10, 24, 30, 33, 17, 29, 10, 50, 18, 34, 15, 40, 13, 32, 36, 32, 27, 35, 17, 41, 18, 36, 20, 41, 35, 51, 29, 27, 44, 43, 15, 32, 29, 54, 14, 45.

Form a frequency table with a grouping of 10 – 20, 20 – 30, 30 – 40 and so on (class 10 – 20 means including 10 but excluding 20, class 20 – 30 means including 20 but excluding 30).

Solution. The grouped frequency table for the given data is:

Classes	*Tally marks*	*Frequency*
10 – 20	~~IIII~~ ~~IIII~~ I	11
20 – 30	~~IIII~~ ~~IIII~~ III	13
30 – 40	~~IIII~~ ~~IIII~~ III	13
40 – 50	~~IIII~~ IIII	9
50 – 60	IIII	4
Total		**50**

Class limits and types of frequency distribution

In example 1, for the class interval 1 – 5, 1 is the **lower limit** and 5 is the **upper limit**. If x is a member of this class, then $1 \le x \le 5$. Similarly, 6 is the *lower limit* and 10 is the *upper limit* of the class 6 – 10. In this example, the classes are non-overlapping but discontinuous. Such a frequency distribution is called **discrete** (or **inclusive**) **distribution**. In this distribution, the upper limit of one class does not coincide with the lower limit of the next class.

In example 2, for the class 1 – 10, 1 is the **lower limit** and 10 is the **upper limit**. If x is a member of this class, then $1 \le x < 10$. Similarly, 10 is the **lower limit** and 20 is the **upper limit** of the class 10 – 20. In this example, the classes are non-overlapping but continuous. Such a frequency distribution is called **continuous** (or **exclusive**) **distribution**. In this distribution, the upper limit of one class coincides with the lower limit of the next class.

Converting discrete distribution to continuous distribution

If we measure height, weight and time, there may be fractions of a metre, kilogram and hour respectively, therefore, we need continuous distribution.

To convert discrete classes into continuous classes, we require some *adjustment*.

$$\text{Adjustment factor} = \frac{\text{lower limit of one class} - \text{upper limit of previous class}}{2}.$$

Subtract the adjustment factor from all the lower limits and add the adjustment factor to all the upper limits.

In example 1, adjustment factor $= \frac{6-5}{2} = \frac{1}{2} = 0{\cdot}5$

Continuous frequency distribution table for example 1 is:

Classes before adjustment	*Classes after adjustment*	*Tally marks*	*Frequency*
1 – 5	0·5 – 5·5	II	2
16 – 10	5·5 – 10·5	~~IIII~~ II	7
11 – 15	10·5 – 15·5	~~IIII~~ III	8
16 – 20	15·5 – 20·5	~~IIII~~ I	6
21 – 25	20·5 – 25·5	II	2
Total			**25**

True class limits

In a continuous distribution, the class limits are called **true** or **actual class limits**. In a discrete distribution, the class limits obtained after adjustment are the *true* or *actual class limits*. The actual class limits are also called ***class boundaries***.

In discrete distribution, the original (given) class limits are called the ***stated class limits***.

Class size. *The difference between the actual upper limit and the actual lower limit of a class is called its* ***class size***.

In example 2, class size of the class 10 – 20 = 20 – 10 = 10.

In example 1, class size of the class 1 – 5 = 5·5 – 0·5 = 5.

Class mark. *The* ***class mark*** *of a class is the value midway between its actual lower limit and actual upper limit.*

In example 2, class mark of the class 10 – 20 = $\frac{10+20}{2}$ = 15.

In example 1, class mark of the class 1 – 5 = $\frac{0.5+5.5}{2}$ = 3.

Note In discrete distribution, the class mark of a class is also the value mid-way between its stated class limits, thus in example 1, the class mark of the class 1 – 5 = $\frac{1+5}{2}$ = 3.

Remarks

1. If the classes are of equal size, then class size = difference between two successive class marks.
2. If the classes are of equal size (width) and h is the size of each class and m is mid-value (class mark) of a class, then

   lower limit of the class = its mid-value – half the width of class

   $= m - \frac{h}{2}$ and

   upper limit of the class = its mid-value + half the width of class

   $= m + \frac{h}{2}$.

Cumulative frequency and cumulative frequency table

The sum of frequencies of all the previous classes and that particular class is called the ***cumulative frequency*** of the class.

The cumulative frequency table for example 2 is:

Classes	*Tally marks*	*Frequency*	*Cumulative frequency*
10 – 20	~~IIII~~ ~~IIII~~ I	11	11
20 – 30	~~IIII~~ ~~IIII~~ III	13	24 (11 + 13)
30 – 40	~~IIII~~ ~~IIII~~ III	13	37 (24 + 13)
40 – 50	~~IIII~~ IIII	9	46 (37 + 9)
50 – 60	IIII	4	50 (46 + 4)

20.3.1 Formation of classes from a given raw data

We condense the given raw data into classes (or groups) as follows:

1. Find the *range i.e.* the difference between the maximum and minimum observations. Decide about the number of classes (usually between 5 to 10). In general, the size (width) is a convenient whole number immediately greater than the quotient obtained by dividing the range by the number of classes.
2. Classes should be non-overlapping and continuous.
3. There should be no gaps between classes.
4. As far as possible, classes should be of the same size.
5. Open ended classes such as less than 5 or greater than 9 should be avoided.
6. Limits of each class should be so chosen that there is no ambiguity as to which class a particular observation of the given data belongs to.

We illustrate the above procedure with the help of the following examples.

Illustrative Examples

Example 1 The electricity bills (in rupees) of 40 houses in a locality are given below. Construct a grouped frequency distribution table:

78, 87, 81, 52, 59, 65, 101, 108, 115, 95, 98, 65, 62, 121, 128, 63, 76, 84, 89, 91, 65, 101, 95, 81, 87, 105, 129, 92, 75, 105, 78, 72, 107, 116, 127, 100, 80, 82, 61, 118.

Solution. Here maximum = 129 and minimum = 52,

$\therefore$ range = 129 − 52 = 77.

Let us form 8 classes each of size 10.

Since we want to include 129 in the last class, 130 is the upper limit of the last class, so the lower limit of the first class is 50.

The grouped frequency distribution table of the given data is:

Classes	*Tally marks*	*Frequency*
50 – 60	II	2
60 – 70	~~IIII~~ I	6
70 – 80	~~IIII~~	5
80 – 90	~~IIII~~ III	8
90 – 100	~~IIII~~	5
100 – 110	~~IIII~~ II	7
110 – 120	III	3
120 – 130	IIII	4
Total		**40**

Example 2 The heights of 240 students of a school are measured and tabulated as below:

Height (in cm)	below 100	below 110	below 120	below 130	below 140	below 150
No. of students	12	30	65	180	218	240

Construct a frequency distribution table for the above data. Also answer the following:

(*i*) How many students have atleast 1 m height but less than 120 cm height?

(*ii*) How many students have atleast 130 cm height?

Solution. The frequency distribution table for the given data is:

Class intervals (Height in cm)	*Frequency (No. of students)*	
0 – 100	12	
100 – 110	18	(= 30 − 12)
110 – 120	35	(= 65 − 30)
120 – 130	115	(= 180 − 65)
130 – 140	38	(= 218 − 180)
140 – 150	22	(= 240 − 218)

(*i*) 53 (= sum of the frequencies of the class intervals 100 – 110 and 110 – 120)

(*ii*) 60 (= sum of the frequencies of the class intervals 130 – 140 and 140 – 150).

Example 3 If m is the mid-point and l is the upper limit of a class in a continuous frequency distribution, then what is the lower limit of the class?

Solution. As l is the upper limit and m is the mid-point of a class in a continuous frequency distribution,

so half of the width of the class = upper limit – mid-point = $l - m$.

Let x be the lower limit of this class, then

$$x = \text{mid-point} - \frac{1}{2} \text{ of width of this class}$$

$$= m - (l - m) = 2m - l.$$

Example 4 Prepare a continuous grouped frequency distribution from the following data:

Mid-points	5	15	25	35	45
Frequency	4	8	13	12	6

Also find the size of class intervals.

Solution. As the mid-points (class marks) of classes are 5, 15, 25, 35 and 45 which are at equal gaps, so the classes are of equal size.

Therefore, size of class = difference between two consecutive mid-points

$$= 15 - 5 = 10.$$

Half of class size = $\frac{10}{2} = 5$.

Therefore, the class limits of the first class are : lower limit = mid-point – half of class size and upper limit = mid-point + half of class size *i.e.* 5 – 5 and 5 + 5 *i.e.* 0 and 10. Similarly, we find the class limits for all other classes. Thus, the classes are 0 – 10, 10 – 20, 20 – 30, 30 – 40 and 40 – 50. The continuous distribution of the given data is:

Classes	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	4	8	13	12	6

EXERCISE

1 State which of the following variables are continuous and which are discrete:

(*i*) marks scored (out of 50) in a test.

(*ii*) daily temperature of your city.

(*iii*) sizes of shoes.

(*iv*) distance travelled by a man.

(*v*) time.

2 Using class intervals 0 –4, 5 –9, 10 –14, ... construct the frequency distribution for the following data :

13, 6, 10, 5, 11, 14, 2, 8, 15, 16, 9, 13, 17, 11, 19, 5, 7, 12, 20, 21, 18, 1, 8, 12, 18.

3 Given below are the marks obtained by 27 students in a test :

21, 3, 28, 38, 6, 40, 20, 26, 9, 8, 14, 18, 20, 16, 17, 10, 8, 5, 22, 27, 34, 2, 35, 31, 16, 28, 37.

(i) Using the class intervals 1 – 10, 11 – 20 etc. construct a frequency table.

(ii) State the range of these marks.

(iii) State the class mark of the third class of your frequency table.

4 Explain the meaning of the following terms :

(i) variate (ii) class size (iii) class mark

(iv) class limits (v) true class limits (vi) frequency of a class

(vii) cumulative frequency of a class.

5 Fill in the blanks :

(i) The number of observations in a particular class is called ..... of the class.

(ii) The difference between the class marks of two consecutive classes is the ..... of the class.

(iii) The range of the data 16, 19, 23, 13, 11, 25, 18 is ..... .

(iv) The mid-point of the class interval is called its ..... .

(v) The class mark of the class 4 – 9 is ..... .

6 The marks obtained (out of 50) by 40 students in a test are given below:

28, 31, 45, 03, 05, 18, 35, 46, 49, 17, 10, 28, 31, 36, 40, 44, 47, 13, 19, 25, 24, 31, 38, 32, 27, 19, 25, 28, 48, 15, 18, 31, 37, 46, 06, 01, 20, 10, 45, 02.

(i) Taking class intervals 1 – 10, 11 – 20, ..., construct a tally chart and a frequency distribution table.

(ii) Convert the above distribution to continuous distribution.

(iii) State the true class limits of the third class.

(iv) State the class mark of the fourth class.

7 Use the adjoining table to find:

(i) upper and lower limits of fifth class.

(ii) true class limits of the fifth class.

(iii) class boundaries of the third class.

(iv) class mark of the fourth class.

(v) width of sixth class.

Class	*Frequency*
28 – 32	5
33 – 37	8
38 – 42	13
43 – 47	9
48 – 52	7
53 – 57	5
58 – 62	2

8 The marks of 200 students in a test were recorded as follows:

Marks %	10–19	20–29	30–39	40–49	50–59	60–69	70–79	80–89
No. of students	7	11	20	46	57	37	15	7

Draw the cumulative frequency table.

9 Given below are the marks secured by 35 students in a test:

41, 32, 35, 21, 11, 47, 42, 00, 05, 18, 25, 24, 29, 38, 30, 04, 14, 24, 34, 44, 48, 33, 36, 38, 41, 46, 08, 34, 39, 11, 13, 27, 26, 43, 03.

Taking class intervals 0 – 10, 10 – 20, 20 – 30 ..., construct frequency as well as cumulative frequency distribution table. Find the number of students obtaining below 20 marks.

10 The marks out of 100 of 50 students in a test are given below:

5	35	6	35	18	36	12	36	85	32
20	36	22	38	24	50	22	39	74	31
25	54	25	64	25	70	28	66	58	25
29	72	31	82	31	84	31	82	37	21
32	84	32	92	35	95	34	92	35	5

(*i*) Taking a class interval of size 10, construct a frequency as well as cumulative frequency table for the given data.

(*ii*) Which class has the largest frequency?

(*iii*) How many students score less than 40 marks?

(*iv*) How many students score first division (60% or more) marks?

11 Construct the frequency distribution table from the following data:

Ages (in years)	below 4	below 7	below 10	below 13	below 16
No. of children	7	38	175	248	300

State the number of children in the age group 10 – 13.

12 Rewrite the following cumulative frequency distribution into frequency distribution:

less than or equal to 10	2
less than or equal to 20	7
less than or equal to 30	18
less than or equal to 40	32
less than or equal to 50	43
less than or equal to 60	50

13 The water bills (in rupees) of 32 houses in a locality are given below. Construct a frequency distribution table with a class size of 10.

80, 48, 52, 78, 103, 85, 37, 94, 72, 73, 66, 52, 92, 85, 78, 81, 64, 60, 75, 78, 108, 63, 71, 54, 59, 75, 100, 103, 35, 89, 95, 73.

14 The maximum temperatures (in degree celsius) for Delhi for the month of April, 2014, as reported by the Meteorological Department, are given below:

27·4, 28·3, 23·9, 23·6, 25·4, 27·5, 28·1, 28·4, 30·5, 29·7, 30·6, 31·7, 32·2, 32·6, 33·4, 35·7, 36·1, 37·2, 38·4, 40·1, 40·2, 40·5, 41·1, 42·0, 42·1, 42·3, 42·4, 42·9, 43·1, 43·2.

Construct a frequency distribution table.

15 (*i*) The class marks of a distribution are 94, 104, 114, 124, 134, 144 and 154. Determine the class size and the class limits of the fourth class.

(*ii*) The class marks of a distribution are 9·5, 16·5, 23·5, 30·5, 37·5 and 44·5. Determine the class size and the class limits of the third class.

SUGGESTED VIDEOS

1. https://www.youtube.com/watch?v=wWenULjri40
2. https://www.youtube.com/watch?v=zjHfAhcU6kE