ESTD. 2001

LEARN TO LEAD

STEPPING STONE SCHOOL (HIGH)

CLASS: IX

SUBJECT - MATH

TOPIC : WORKSHEET # 14

Dated : 08.06.2020

STEPPING STONE SCHOOL (HIGH)

MATHEMATICS

CLASS: 9

WORKSHEET NO 14 dt:

TOPIC: TRIANGLES.

SUB TOPIC: CONGRUENCY.

CONGRUENT TRIANGLES

Two triangles are said to be **congruent** to each other, if on placing one over the other, they exactly coincide.

In fact, two triangles are congruent, if they have exactly the *same shape* and the *same size. i.e.*, all the angles and all the sides of one triangle are equal to the corresponding angles and the corresponding sides of the other triangle each to each.

Triangles with same shape means : Angles of one triangle are equal to angles of other triangle each to each.

Triangles with same size means : Sides of one triangle are equal to sides of other triangle each to each.

The given figure shows two triangles ABC and DEF such that :

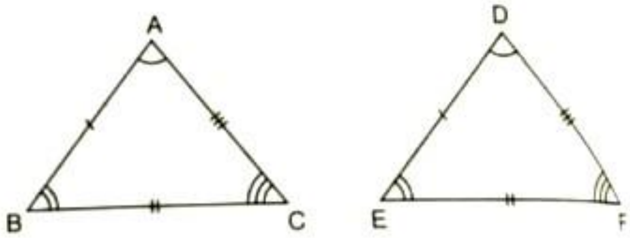
(i) $\angle A = \angle D$; $\angle B = \angle E$ and $\angle C = \angle F$.

(ii) AB = DE; BC = EF and AC = DF.

$\therefore$ Δ ABC is congruent to Δ DEF

and we write : **Δ ABC $\cong$ Δ DEF.**

The symbol $\cong$ is read as "*is congruent to*".

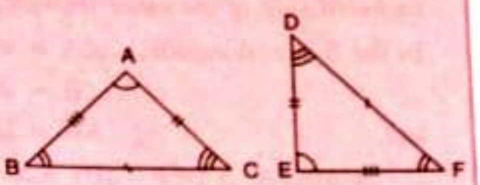


1. Congruent figures (triangles) always coincide by *superposition i.e.* by placing one figure over the other.
2. In congruent triangles, the sides and the angles that *coincide* by *superposition* are called *corresponding sides* and *corresponding angles.*
3. The corresponding sides lie *opposite* to the *equal angles* and corresponding angles lie *opposite* to the *equal sides.*

In the figure alongside, Δ ABC ≅ Δ EFD.

Since, ∠A = ∠E, therefore the side opposite to ∠A and the side opposite to ∠E are corresponding sides *i.e.*, BC and DF are corresponding sides.

Similarly, AB and EF are corresponding sides as ∠C = ∠D.



Also, AC and DE are corresponding sides.

Conversely, as side AB = side EF, therefore, angles opposite to these sides *i.e.* ∠C and ∠D are the corresponding angles and so on.

4. Corresponding Parts of Congruent Triangles are also Congruent.

Abbreviated as : **C.P.C.T.C.**

CONDITIONS FOR CONGRUENCY OF TRIANGLES

1. *If two sides and the included angle of one triangle are equal to two sides and the included angle of the other triangle, the triangles are congruent.*

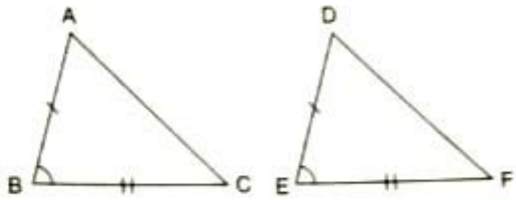
Abbreviated as : **S.A.S.**

In the figure alongside,

AB = DE;

BC = EF and ∠B = ∠E.

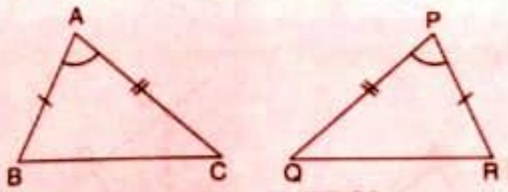
∴ Δ ABC ≅ Δ DEF [By S.A.S.]



Triangles will be congruent only when the equal angles are the included angles.

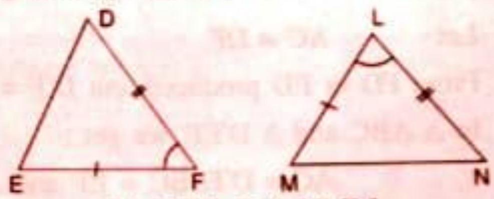
In each of the following figures, triangles are congruent by S.A.S. :

(i)



[Δ ABC ≅ Δ PRQ]

(ii)



[Δ DEF ≅ Δ NML]

Proof :

Let triangles ABC and DEF have AB = DE, AC = DF and ∠A = ∠D.

Required to prove : Δ ABC ≅ Δ DEF

Place Δ ABC over Δ DEF such that A falls on D and B falls on E.

Since, ∠A = ∠D, so AC will fall on DF.

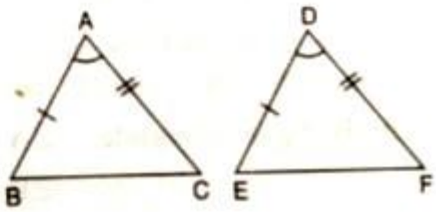
∵ AC = DF and A falls on D

∴ C will fall on F

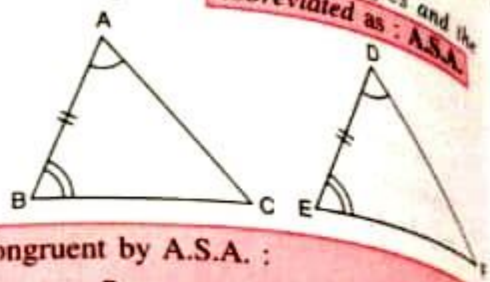
Since, A falls on D, B falls on E and C falls on F

⇒ Δ ABC covers Δ DEF completely

⇒ **Δ ABC ≅ Δ DEF**



2. *If two angles and the included side of one triangle are equal to two angles and the included side of the other triangle, the triangles are congruent.* **Abbreviated as : A.S.A.**



In the figure alongside, ∠A = ∠D,

∠B = ∠E

and, AB = DE

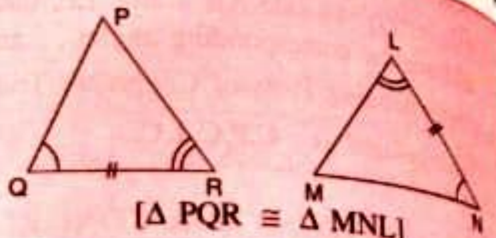
∴ Δ ABC ≡ Δ DEF. [By A.S.A.]

In each of the following figures, triangles are congruent by A.S.A. :

(i)

[Δ ABC ≡ Δ FDE]

(ii)

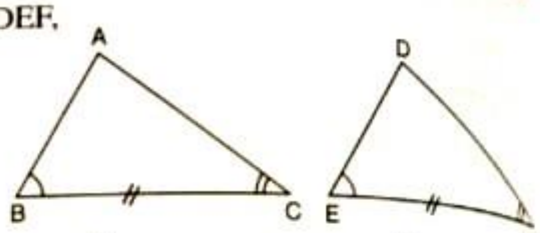


[Δ PQR ≡ Δ MNL]

Proof :

Let triangles ABC and DEF have ∠ABC = ∠DEF, ∠ACB = ∠DFE and BC = EF

Required to prove : Δ ABC ≅ Δ DEF



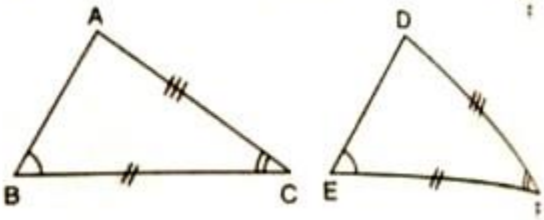
Case 1 :

Let AC = DF

∵ BC = EF (given)

and ∠C = ∠F (given)

∴ **Δ ABC ≅ Δ DEF** (by S.A.S.)



Case 2 :

Let AC ≠ DF

From FD or FD produced, cut D'F = AC

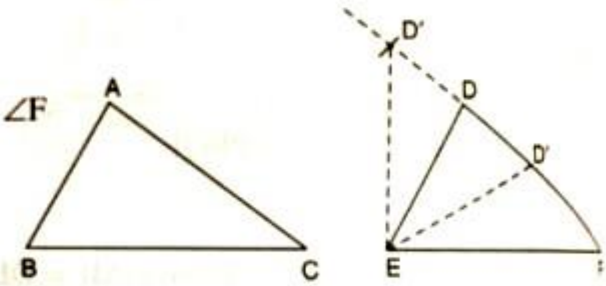
In Δ ABC and Δ D'EF, we get :

AC = D'F, BC = EF and ∠C = ∠F

⇒ Δ ABC ≅ Δ D'EF (by S.A.S.)

⇒ ∠B = ∠D'EF

⇒ ∠D'EF = ∠DEF



Now AC = D'F = DF, BC = EF and ∠C = ∠F

This is possible only if D and D' coincide.

⇒ Δ ABC ≅ Δ DEF (by S.A.S.)

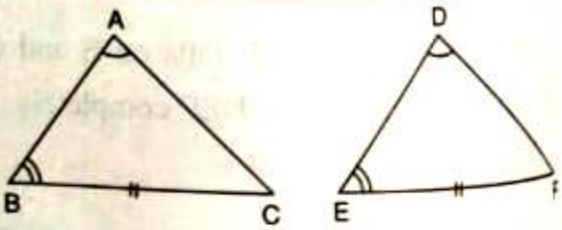
3. *If two angles and one side of one triangle are equal to two angles and the corresponding side of the other triangle, the triangles are congruent.* **Abbreviated as : A.A.S.**

In the figure alongside, ∠A = ∠D,

∠B = ∠E

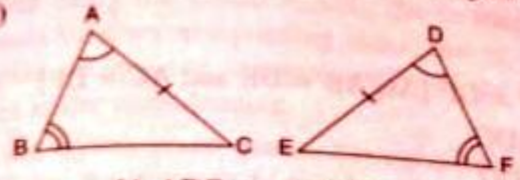
and, BC = EF

∴ Δ ABC ≅ Δ DEF. [By A.A.S.].



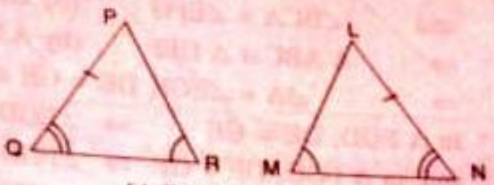
In each of the following figures, triangles are congruent by A.A.S. :

(i)



[Δ ABC ≅ Δ DFE]

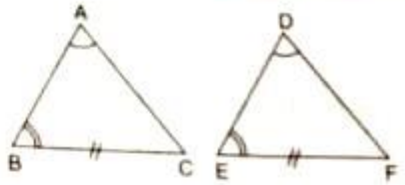
(ii)



[Δ PQR ≅ Δ LNM]

Proof

Let in Δ ABC and Δ DEF; ∠A = ∠D, ∠B = ∠E and BC = EF



Required to prove : Δ ABC ≅ Δ DEF

Since, $\angle A + \angle B + \angle C = 180^\circ$

and $\angle D + \angle E + \angle F = 180^\circ$

⇒ $\cancel{\angle A} + \cancel{\angle B} + \angle C = \cancel{\angle D} + \cancel{\angle E} + \angle F$ [As, ∠A = ∠D and ∠B = ∠E]

⇒ $\angle C = \angle F$

Now, ∠B = ∠E, ∠C = ∠F and BC = EF

⇒ **Δ ABC ≅ Δ DEF** (by A.S.A)

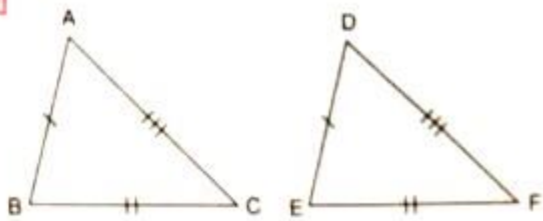
4. *If three sides of one triangle are equal to three sides of the other triangle, each to each, the triangles are congruent.* **Abbreviated as : S.S.S.**

In the figure alongside, AB = DE,

BC = EF

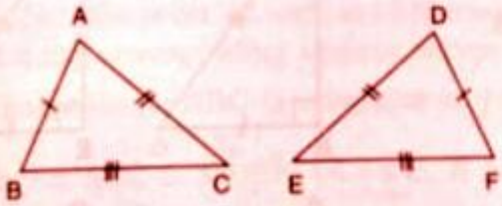
and, AC = DF

∴ Δ ABC ≅ Δ DEF. [By S.S.S.].



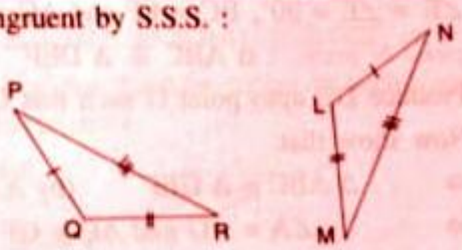
In each of the following figures, triangles are congruent by S.S.S. :

(i)



[Δ ABC ≅ Δ DFE]

(ii)



[Δ PQR ≅ Δ NLM]

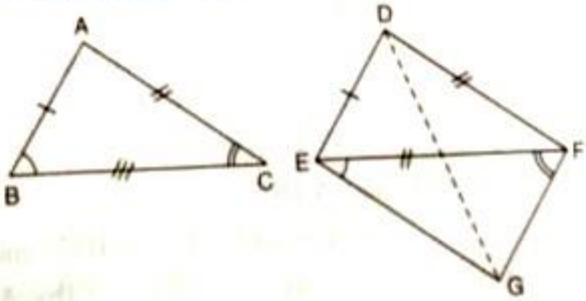
Proof :

Let Δ ABC and Δ DEF have AB = DE, AC = DF and BC = EF.

Required to prove : Δ ABC ≅ Δ DEF

Let BC be the longest side of triangle ABC and so EF is the longest side of triangle DEF.

Draw EG so that ∠FEG = ∠CBA and ∠EFG = ∠BCA. Join D and G.



Now in Δ ABC and Δ GEF,

BC = EF (given)

and $\angle CBA = \angle FEG$ (by construction)

$\angle BCA = \angle EFG$ (by construction)

$\Rightarrow$ $\Delta ABC \cong \Delta GEF$ (by A.S.A.)

$\Rightarrow$ $\angle A = \angle EGF$, DE = GE and DF = GF [As, AB = DE and AC = DF]

In Δ EGD, DE = GE $\Rightarrow$ $\angle EGD = \angle EDG$

and, in Δ FGD, DF = GF $\Rightarrow$ $\angle FGD = \angle FDG$

$\Rightarrow$ $\angle EGD + \angle FGD = \angle EDG + \angle FDG$

$\Rightarrow$ $\angle EGF = \angle EDF$

$\Rightarrow$ $\angle A = \angle EDF$ [$\because \angle EGF = \angle A$]

$\Rightarrow$ $\angle A = \angle D$

In Δ ABC and Δ DEF,

AB = DE,

AC = DF and

$\angle A = \angle D$

$\Rightarrow$ **$\Delta ABC \cong \Delta DEF$** (by S.A.S.)

5. *Two right-angled triangles are congruent, if the hypotenuse and one side of one triangle are equal to the hypotenuse and corresponding side of the other triangle.* **Abbreviated as : R.H.S.**

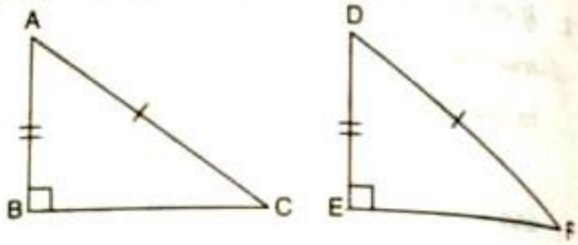
The given figure shows two right-angled triangles ABC and DEF such that :

$\angle B = \angle E = 90°$;

AC = DF

and, AB = DE

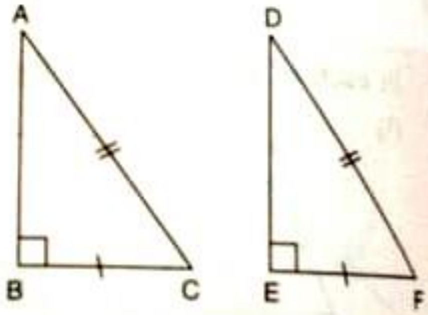
$\therefore$ $\Delta ABC \cong \Delta DEF$. [By R.H.S.]



Proof :

Let in right angled trianlges ABC and DEF,

$\angle B = \angle E = 90°$, BC = EF and AC = DF



Required to prove : $\Delta ABC \cong \Delta DEF$

Produce DE upto point G such that GE = AB.

Now show that

$\Rightarrow$ $\Delta ABC \cong \Delta GEF$ (by A.S.A.)

$\Rightarrow$ $\angle A = \angle G$ and AC = GF

$\Rightarrow$ AC = GF and AC = DF (given)

$\Rightarrow$ DF = GF

$\Rightarrow$ $\angle G = \angle D$

$\Rightarrow$ $\angle A = \angle D$ (as, $\angle A = \angle G$)

In Δ ABC and Δ DEF

$\angle A = \angle D$ (proved above)

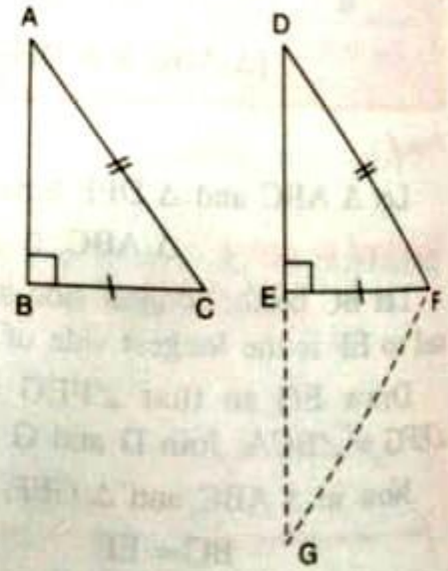
$\angle B = \angle DEF = 90°$

$\Rightarrow$ $\angle C = \angle DFE$

In Δ ABC and Δ DEF,

$\angle A = \angle D$, $\angle C = \angle DFE$ and AC = DF

$\Rightarrow$ **$\Delta ABC \cong \Delta DEF$** (by A.S.A.)



CONGRUENCY OF TRIANGLES

Ex.1. Which of the following pairs of triangles are congruent? Give reason.

(a) $\Delta ABC : BC = 4$ cm, $CA = 5$ cm, $\angle C = 70^\circ$

$\Delta PQR : PQ = 4$ cm, $QR = 5$ cm, $\angle Q = 70^\circ$

(b) $\Delta ABC : AB = 4$ cm, $BC = 5$ cm, $\angle B = 70^\circ$

$\Delta PQR : PQ = 4$ cm, $RP = 5$ cm, $\angle R = 70^\circ$

(c) $\Delta ABC : AB = 5$ cm, $BC = 7$ cm, $CA = 9$ cm

$\Delta PQR : PQ = 7$ cm, $QR = 5$ cm, $RP = 9$ cm

Sol. (a) $\Delta ABC : BC = 4$ cm, $CA = 5$ cm, $\angle C = 70^\circ$

$\Delta PQR : PQ = 4$ cm, $QR = 5$ cm, $\angle Q = 70^\circ$

Yes, $\Delta BCA \cong \Delta PQR$ [SAS condition of congruency is satisfied]

(b) $\Delta ABC : AB = 4$ cm, $BC = 5$ cm, $\angle B = 70^\circ$

$\Delta PQR : PQ = 4$ cm, $RP = 5$ cm, $\angle R = 70^\circ$

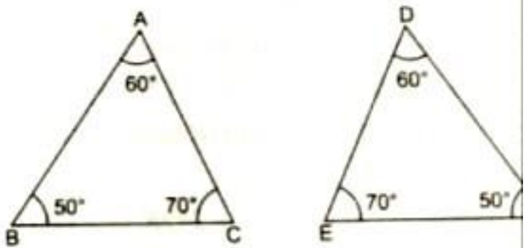
ΔABC is not congruent to ΔPQR, because two sides are equal but included angle is not same

(c) $\Delta ABC : AB = 5$ cm, $BC = 7$ cm, $CA = 9$ cm

$\Delta PQR : PQ = 7$ cm, $QR = 5$ cm, $RP = 9$ cm

$\Delta ABC \cong \Delta RQP$ [SSS condition of congruency is satisfie

Ex.2. ABC and DEF are two triangles in which $AB = DF$, $\angle ACB = 70^\circ$, $\angle ABC = 50^\circ$, $\angle DEF = 70^\circ$ an $\angle EDF = 60^\circ$. Prove that the two triangles are congruent.



Sol. In ΔABC, $\angle BAC = 60^\circ$ and in ΔDEF, $\angle DFE = 50^\circ$

($\because$ Sum of angles of a triangle is 180°)

In ΔABC and ΔDEF,

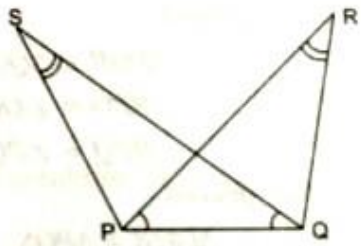
$AB = DF$ [Given]

$\angle BAC = \angle EDF = 60^\circ$

$\angle ACB = \angle DEF = 70^\circ$ [Given]

Hence, $\Delta ABC \cong \Delta DFE$ [$\because$ AAS-condition of congruency is satisfi

Ex.3. In the given figure, $\angle R = \angle S$ and $\angle RPQ = \angle PQS$. Prove that $PS = QR$.



Sol. Given: $\angle R = \angle S$ and $\angle RPQ = \angle PQS$

To prove: $PS = QR$

Proof: In ΔPQS and ΔPQR, we have

$PQ = PQ$ [Comm

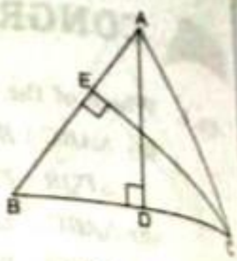
$\angle PSQ = \angle PRQ$ [Gi

$\angle RPQ = \angle PQS$ [Gi

Hence, $\Delta PQS \cong \Delta PQR$ [A

$\therefore$ $PS = QR$ [C

Ex.4. In the given figure, $AB = BC$, $AD \perp BC$, $CE \perp AB$. Prove that $AD = CE$.



Sol. Given: $AB = BC$, $AD \perp BC$ and $CE \perp AB$ [Given]

To prove: $AD = CE$

Proof: $AB = BC$ [Given]

$\therefore$ In ΔABC, $\angle ACB = \angle CAB$ [$\because$ opposite $\angle$s of equal sides are equal]

In ΔACE and ΔADC, $\angle EAC = \angle ACD$ [$\angle ACB = \angle BAC$]

$\angle CEA = \angle ADC = 90°$

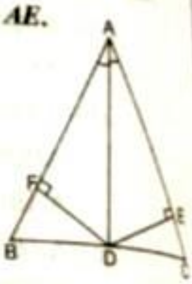
$AC = AC$ [AC is common]

Hence, $\Delta AEC \cong \Delta ADC$ [AAS]

$\therefore$ $AD = CE$ [CPCT]

Hence proved.

Ex.5. In the given figure, AD bisects $\angle A$, $DE \perp CA$ and $DF \perp AB$. Prove that $AF = AE$.



Sol. Given: AD is the bisector of $\angle BAC$. $DE \perp CA$ and $DF \perp AB$

To prove: $AF = AE$

Proof: In ΔAFD and ΔAED

$AD = AD$ [Common]

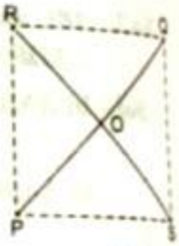
$\angle FAD = \angle DAE$ [$\because$ AD is bisector of $\angle FAE$]

$\angle AFD = \angle AED = 90°$ [$\because$ $DF \perp AB$ and $DE \perp AC$]

$\therefore$ $\Delta AFD \cong \Delta AED$ [$\because$ AAS-conditions of congruency is satisfied]

Hence, $AF = AE$ [CPCT]

Ex.6. Two line segments PQ and RS bisect each other at O. Prove that:



(a) $PR = QS$ **(b) $\angle RPQ = \angle PQS$**

(c) $PS \parallel RQ$ **(d) $PS = RQ$**

Sol. Given: PQ and RS bisect each other at O.

To prove: (a) $PR = QS$ (b) $\angle RPQ = \angle PQS$ (c) $PS \parallel RQ$ (d) $PS = RQ$

Proof: (a) In ΔPOR and ΔQOS, we have joined PR, PS, RQ and QS, $OR = OS$

$PO = OQ$ [Given]

$\angle POR = \angle QOS$ [Vertically opposite $\angle$s]

Hence, $\Delta POR \cong \Delta QOS$ [$\because$ SAS-condition of congruency is satisfied]

$\therefore$ $PR = QS$ [CPCT]

Hence proved.

(b) $\because$ $\Delta POR \cong \Delta QOS$

Hence, $\angle RPO = \angle OQS$ [CPCT]

$\Rightarrow$ $\angle RPQ = \angle PQS$

Hence proved.

(c) Similarly, $\Delta QOR \cong \Delta POS$ [$\because$ SAS-condition of congruency is satisfied]

$\therefore$ $\angle OQR = \angle OPS$ [CPCT]

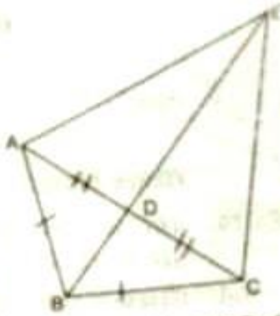
Hence, $PS \parallel QR$ [Pair of alternate interior $\angle$s are equal]

(d) $\because$ $\Delta QOR \cong \Delta POS$

Hence, $PS = RQ$ [CPCT]

Hence proved.

Ex.7. In the given figure, AB = BC and AD = CD. Prove that:
(a) ∠ADE = a right angle **(b) AE = EC.**



Sol. Given: $AB = BC$ and $AD = DC$

To prove: (a) $\angle ADE = 90°$ (b) $AE = EC$

Proof: In ΔABD and ΔBCD, we have

$AB = BC$ [Given]

$AD = DC$ [Given]

$BD = BD$ [Common]

Hence, $\Delta ABD \cong \Delta BCD$ [∵ SSS-condition of congruency is satisfied]

(a) $\angle ADB = \angle CDB$ [CPCT]

But, $\angle ADB$ and $\angle CDB$ form a linear pair of angles.

$\angle ADB = \angle CDB = 90°$

$\angle ADE = \angle CDB = 90°$ [Vertically opposite ∠s]

$\angle ADE = 90°$ **Hence proved.**

(b) In ΔADE and ΔCDE, we have DE is common

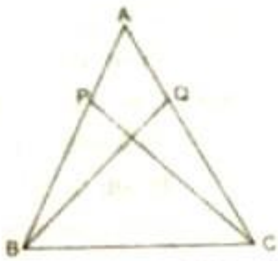
$AD = DC$ [Given]

$\angle ADE = \angle EDC = 90°$

So $\Delta ADE \cong \Delta CDE$ [∵ ASA-condition of congruency is satisfied]

Hence, $AE = EC$. **Hence proved.**

Ex.8. In the given figure, AB = AC and AP = AQ. Prove that:
(a) ΔAPC ≅ ΔAQB. **(b) ΔBPC ≅ ΔCQB.**



Sol. Given: $AB = AC$ and $AP = AQ$

To prove: (a) $\Delta APC \cong \Delta AQB$ (b) $\Delta BPC \cong \Delta CQB$

Proof: $AB - AP = AC - AQ$ [∵ $AP = AQ$]

i.e. $PB = QC$

(a) In ΔAPC and ΔAQB, we have

$AP = AQ$ [Given]

$\angle PAC = \angle BAQ$ [Common]

$AB = AC$ [Given]

Hence, $\Delta APC \cong \Delta AQB$ [∵ SAS-condition of congruency is satisfied]

$CP = BQ$ [CPCT]

Hence proved.

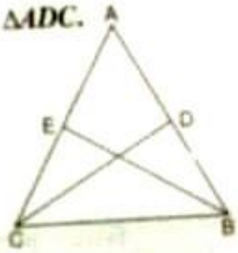
(b) In ΔBPC and ΔCQB, we have BC is common

$PB = CQ$ and $CP = BQ$ [Proved above]

∴ $\Delta BPC \cong \Delta CQB$ [∵ SSS-condition of congruency is satisfied]

Hence proved.

Ex.9. In the given figure, it is given that AE = AD and BD = CE. Prove that ΔAEB ≅ ΔADC.



Sol. $AE = AD$ [Given]

and $BD = CE$ [Given]

In ΔAEB and ΔADC,

$AE = AD$ [Given]

$CE = BD$ [Given]

$\therefore \quad AE + CE = AD + BD$

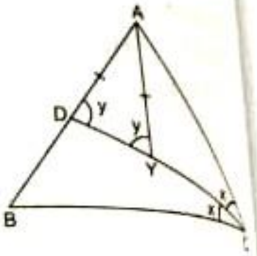
$AC = AB$ [Common]

Also, $\angle EAB = \angle DAC$

$\Delta AEB \cong \Delta ADC$ ($\because$ *SAS* condition of congruency is satisfied)

Hence proved.

Ex.10. *ABC* is a triangle. The bisector of the angle *BCA* meets *AB* in *D*. A point *Y* lies on *CD* such that *AD* = *AY*. Prove that $\angle CAY = \angle ABC$.



Sol. Given: *CD* is the bisector of $\angle ACB$. $AD = AY$

To prove: $\angle CAY = \angle ABC$

Proof: Let $\angle ACD = \angle BCD = x$

and $\angle ADY = \angle AYD = y$

In ΔADC, $y + x + \angle DAC = 180°$ [Sum of all $\angle$s of a Δ is 180°]

$\angle DAC = 180° - (x + y)$

Also in ΔABC, $\angle ABC + 2x + 180° - (x + y) = 180°$

$\angle ABC = y - x$ ...(i)

$\angle AYC = 180° - y$ [Linear pair]

In ΔAYC, $\angle YAC = 180° - [x + (180° - y)]$

$\angle YAC = y - x$ ...(ii)

From equation (*i*) and (*ii*), $\angle CAY = \angle ABC$

Hence proved.

Ex.11. *P* is any point in the ΔABC such that the perpendicular drawn from *P* on *AB* and *AC* are equal. Prove that *AP* is the bisector of $\angle BAC$.

Sol. Given: *P* is a point in ΔABC. $PQ \perp AB$ and $PR \perp AC$. Also $PQ = PR$

To prove: $\angle BAP = \angle CAP$

Proof: In ΔAPQ and ΔAPR, we have

AP is common.

$PQ = PR$ [Given]

$\angle AQP = \angle ARP = 90°$

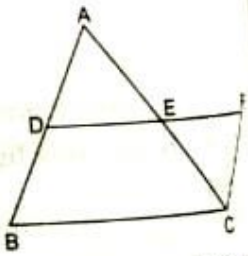
Hence, $\Delta APQ \cong \Delta APR$ [$\because$ *RHS*-condition of congruency is satisfied]

$\angle QAP = \angle RAP$ [*CPCT*]

$\therefore$ *AP* is bisector of $\angle BAC$

Hence proved.

Ex.12. *ABC* is a triangle. *D* and *E* are mid-points of the sides *AB* and *AC* respectively. *DE* is produced to *F*, such that *DE* = *EF*. Prove that $\Delta ADE \cong \Delta EFC$.



Sol. Given: *D* and *E* are mid-points of *AB* and *AC* respectively.

DE is produced to *F* such that $DE = EF$

To prove: $\Delta ADE \cong \Delta EFC$

Proof: In ΔADE and ΔEFC

$DE = EF$ [Given]

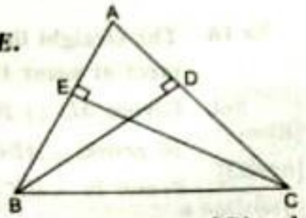
$\angle AED = \angle FEC$ [Vertically opposite $\angle$s]

$AE = EC$ [Given]

$\Delta ADE \cong \Delta EFC$ [$\because$ *SAS*-condition of congruency is satisfied]

Hence proved.

Ex.13. In the given figure, $AB = AC$. $BD \perp AC$, $CE \perp AB$. Prove that $BD = CE$.



Sol. Given: $BD \perp AC$ and $CE \perp AB$. $AB = AC$

To Prove: $BD = CE$

Proof: In ΔABD and ΔACE, we have

$AB = AC$ [Given]

$\angle A$ is common

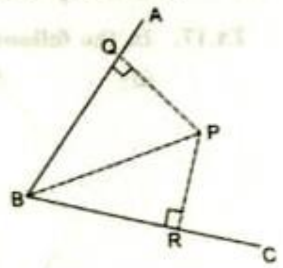
$\angle BDA = \angle AEC = 90°$ [Given]

Hence, $\Delta ABD \cong \Delta ACE$ [$\because$ *AAS*-condition of congruency is satisfied]

$\therefore$ $BD = EC$ [*CPCT*]

Hence proved.

Ex.14. Prove that any point on the bisector of an angle is equidistant from the arms of the angle.



Sol. Given: An $\angle ABC$ and P is any point on bisector of $\angle ABC$.

Const. Draw $PQ \perp AB$ and $PR \perp BC$.

To prove: $PQ = PR$

Proof: In ΔBPQ and ΔBPR, we have

BP is common.

$\angle QBP = \angle PBR$ [*BP* is bisector of $\angle ABC$]

$\angle PQB = \angle PRB = 90°$ [Given]

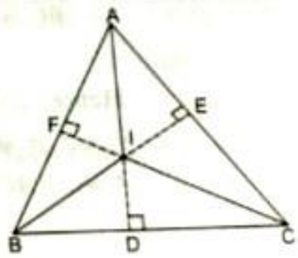
Hence, $\Delta BPQ \cong \Delta BPR$ [$\because$ *AAS*-condition of congruency is satisfied]

$\therefore$ $PQ = PR$ [*CPCT*]

Hence proved.

Ex.15. Prove that the internal bisectors of the angles of a triangle are concurrent.

Sol. Given: ABC is a triangle. BI, CI internal bisectors of $\angle B$ and $\angle C$ meet at I. AI bisects $\angle A$ internally.



To prove: Bisectors of $\angle A$, $\angle B$ and $\angle C$ meet at I.

i.e., they are concurrent.

Const. Draw ID, IE, IF perpendiculars on BC, CA and AB respectively.

Proof: In ΔIBF and ΔIBD.

$\angle IFB = \angle IDB = 90°$ [By construction]

$\angle IBD = \angle IBF$ [*BI* is bisector]

$BI = BI$ is common

$\therefore$ $\Delta IBF \cong \Delta IBD$ [$\because$ *AAS*-condition of congruency is satisfied]

$\therefore$ $IF = ID$ [*CPCT*]

Similarly, in Δs IDC, IEC and in Δs IAF, IAE, we can have,

$IE = ID$, $IF = IE$

In Δs AIF and AIE

$\angle AFI = \angle AEI = 90°$ [By construction]

$AI = AI$

$\therefore$ $\Delta AFI \cong \Delta AEI$ [$\because$ *RHS*-condition of congruency is satisfied]

$\therefore$ $\angle FAI = \angle EAI$ [*CPCT*]

$\therefore$ AI bisects $\angle BAC$

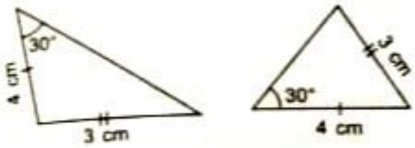
Hence, internal bisectors of the angles of a Δ are concurrent.

Hence proved

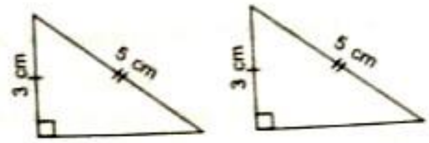
Practice Questions

1. State which of the pair of triangles are congruent. Mention in each case the type of congruency (namely, *SSS*, *SAS*, *AAS*, RHS)

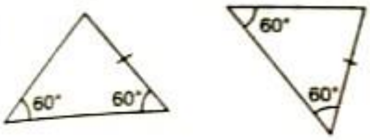
(*a*)



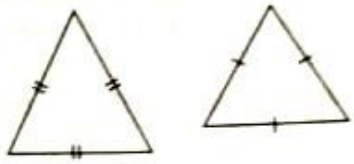
(*b*)



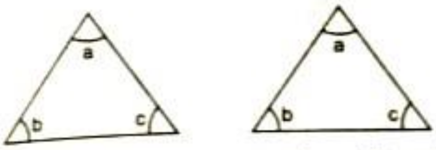
(*c*)



(*d*)



(*e*)



2. In the given fig. (*i*), prove that $AE = BD$.
3. In the given fig. (*ii*), prove that AD and BC bisect each other at O.
4. In the given fig. (*iii*), P is a point in the interior of $\angle LMN$ such that $PA \perp ML$, $PB \perp MN$ and $PA = PB$. Show that P lies on the bisector of $\angle LMN$.

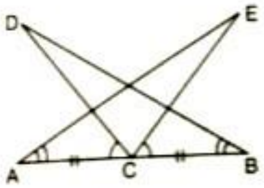


Fig. (*i*)

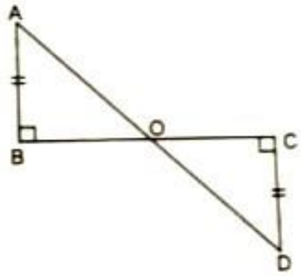


Fig. (*ii*)

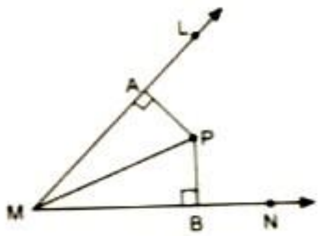


Fig. (*iii*)

5. In the given fig. (*iv*), prove that
   (*a*) $\Delta ABC \cong \Delta ADC$ (*b*) $\angle B = \angle D$ (*c*) $AB = CD$ and $BC = AD$
6. In the given fig. (*v*), OP bisects $\angle P$ and $\angle OQP = \angle ORP$, prove that $\Delta OPQ \cong \Delta OPR$.
7. In the given fig. (*vi*), $ABCD$ is a square and EF is parallel to BD. R is the mid-point of EF. Prove that
   (*a*) $BE = DF$ (*b*) AR bisects $\angle BAD$ (*c*) If AR is produced it will pass through C.

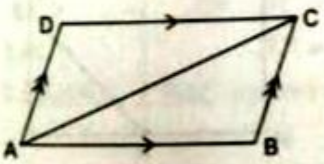


Fig. (*iv*)

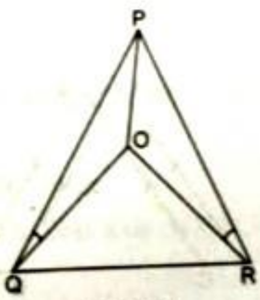


Fig. (*v*)

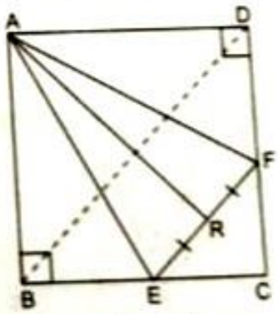


Fig. (*vi*)