CLASS: IX

SUBJECT - MATH

TOPIC: WORKSHEET # 14

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CONGRUENT TRIANGLES

Two triangles are said to be congruent to each other, if on placing one over the other, they exactly coincide.

In fact, two triangles are congruent, if they have exactly the same shape and the same size, i.e., all the angles and all the sides of one triangle are equal to the corresponding angles and the corresponding sides of the other triangle each to each.

Triangles with same shape means: Angles of one triangle are equal to angles of other triangle each to each.

Triangles with same size means: Sides of one triangle are equal to sides of other triangle each to each.

The given figure shows two triangles ABC and DEF such that:

(i) \( \angle A = \angle D; \angle B = \angle E \) and \( \angle C = \angle F \).

(ii) \( AB = DE; BC = EF \) and \( AC = DF \).

\[ \therefore \triangle ABC \text{ is congruent to } \triangle DEF \]

and we write : \( \triangle ABC \cong \triangle DEF \).

The symbol \( \cong \) is read as “is congruent to”.

1. Congruent figures (triangles) always coincide by superposition i.e. by placing one figure over the other.

2. In congruent triangles, the sides and the angles that coincide by superposition are called corresponding sides and corresponding angles.

3. The corresponding sides lie opposite to the equal angles and corresponding angles lie opposite to the equal sides.
In the figure alongside, \( \triangle ABC \equiv \triangle EPD \).

Since, \( \angle A = \angle E \), therefore the side opposite to \( \angle A \) and the side opposite to \( \angle E \) are corresponding sides i.e., \( BC \) and \( DF \) are corresponding sides.

Similarly, \( AB \) and \( EF \) are corresponding sides as \( \angle C = \angle D \).

Also, \( AC \) and \( DE \) are corresponding sides.

Conversely, as side \( AB = \) side \( EF \), therefore, angles opposite to these sides i.e. \( \angle C \) and \( \angle D \) are the corresponding angles and so on.

4. Corresponding Parts of Congruent Triangles are also Congruent.

**Abbreviated as : C.P.C.T.C.**

**CONDITIONS FOR CONGRUENCY OF TRIANGLES**

1. If two sides and the included angle of one triangle are equal to two sides and the included angle of the other triangle, the triangles are congruent.

*Abbreviated as : S.A.S.*

In the figure alongside.

\[
AB = DE;
\]

BC = EF and \( \angle B = \angle E \).

\[
\therefore \triangle ABC \equiv \triangle DEF \quad \text{[By S.A.S.]} \]

**Triangles will be congruent only when the equal angles are the included angles.**

In each of the following figures, triangles are congruent by S.A.S. :

(i) \[
\triangle ABC \equiv \triangle PRQ
\]

(ii) \[
\triangle DEF \equiv \triangle NML
\]

**Proof :**

Let triangles ABC and DEF have \( AB = DE, AC = DF \) and \( \angle A = \angle D \).

**Required to prove :** \( \triangle ABC \equiv \triangle DEF \)

Place \( \triangle ABC \) over \( \triangle DEF \) such that A falls on D and B falls on E.

Since, \( \angle A = \angle D \), so AC will fall on DF.

\[
\therefore AC = DF \text{ and } A \text{ falls on } D
\]

\[
\therefore C \text{ will fall on } F
\]

Since, A falls on D, B falls on E and C falls on F

\[
\Rightarrow \triangle ABC \text{ covers } \triangle DEF \text{ completely}
\]

\[
\Rightarrow \triangle ABC \equiv \triangle DEF
\]
2. If two angles and the included side of one triangle are equal to two angles and the included side of the other triangle, the triangles are congruent. **Abbreviated as:** A.A.S.

In the figure alongside, \( \angle A = \angle D, \) 
\( \angle B = \angle E \) 
\( AB = DE \)

and,
\[ \triangle ABC \equiv \triangle DEF. \] [By A.S.A.]

In each of the following figures, triangles are congruent by A.S.A. :

(i) \( \triangle ABC \equiv \triangle FDE \) 
(ii) \( \triangle PQR \equiv \triangle MNL \)

**Proof:**

Let triangles ABC and DEF have \( \angle ABC = \angle DEF, \) 
\( \angle ACB = \angle DFE \) and \( BC = EF \)

**Required to prove:** \( \triangle ABC \equiv \triangle DEF \)

**Case 1:**

Let \( AC = DF \)
\[ \therefore BC = EF \] [given]
and \( \angle C = \angle F \) [given]
\[ \therefore \triangle ABC \equiv \triangle DEF \] [by S.A.S.]

**Case 2:**

Let \( AC \neq DF \)
From FD or FD produced, cut \( D'F = AC \)
In \( \triangle ABC \) and \( \triangle D'EF, \) we get :
\[ AC = D'F, \ BC = EF \] and \( \angle C = \angle F \)
\[ \Rightarrow \triangle ABC \equiv \triangle D'EF \] [by S.A.S.]
\[ \Rightarrow \angle B = \angle D'EF \]
\[ \Rightarrow \angle D'EF = \angle DEF \]

Now \( AC = D'F = DF, \ BC = EF \) and \( \angle C = \angle F \)
This is possible only if \( D \) and \( D' \) coincide.
\[ \Rightarrow \triangle ABC \equiv \triangle DEF \] [by S.A.S.]

3. If two angles and one side of one triangle are equal to two angles and the corresponding side of the other triangle, the triangles are congruent. **Abbreviated as:** A.A.S.

In the figure alongside, \( \angle A = \angle D, \) 
\( \angle B = \angle E \)

and,
\[ BC = EF \]
\[ \therefore \triangle ABC \equiv \triangle DEF. \] [By A.A.S.]
In each of the following figures, triangles are congruent by A.A.S.:

(i) \[ \triangle ABC \cong \triangle DFE \]

(ii) \[ \triangle PQR \cong \triangle LNM \]

**Proof**

Let \( \triangle ABC \) and \( \triangle DEF \): \( \angle A = \angle D \), \( \angle B = \angle E \) and \( BC = EF \)

**Required to prove**: \( \triangle ABC \cong \triangle DEF \)

Since, \( \angle A + \angle B + \angle C = 180^\circ \)
and \( \angle D + \angle E + \angle F = 180^\circ \)

So, \( \angle A = \angle D \) and \( \angle B = \angle E \)

**Abbreviated as**: S.S.S.

Now, \( \angle B = \angle E \), \( \angle C = \angle F \) and \( BC = EF \)

\[ \triangle ABC \cong \triangle DEF \]  
(by A.S.A)

4. If three sides of one triangle are equal to three sides of the other triangle, each to each, the triangles are congruent **Abbreviated as**: S.S.S.

In the figure alongside, \( AB = DE \), \( BC = EF \), \( AC = DF \)

and, \( \triangle ABC \cong \triangle DEF \)  
[By S.S.S.]

In each of the following figures, triangles are congruent by S.S.S.:

(i) \[ \triangle ABC \cong \triangle DFE \]

(ii) \[ \triangle PQR \cong \triangle NLM \]

**Proof**:

Let \( \triangle ABC \) and \( \triangle DEF \) have \( AB = DE \), \( AC = DF \) and \( BC = EF \).

**Required to prove**: \( \triangle ABC \cong \triangle DEF \)

Let \( BC \) be the longest side of triangle \( ABC \) and so \( EF \) is the longest side of triangle \( DEF \).

Draw \( EG \) so that \( \angle FEG = \angle CBA \) and \( \angle EFG = \angle BCA \). Join \( D \) and \( G \).

Now in \( \triangle ABC \) and \( \triangle GEF \),

\( BC = EF \)  
(given)

Proof continues...
\[ \angle CBA = \angle FEG \quad \text{(by construction)} \]

and
\[ \angle BCA = \angle EFG \quad \text{(by A.S.A.)} \]

\[ \Rightarrow \Delta ABC \cong \Delta GEF \quad \text{(by A.S.A.)} \]

\[ \angle A = \angle GEF, \, DE = GE \, \text{and} \, DF = GF \, [\text{As,} \, AB = DE \, \text{and} \, AC = DF] \]

\[ \Rightarrow \angle A = \angle EGD \quad \text{and, in} \, \Delta GFD, \, DF = GF \Rightarrow \angle FGD = \angle FDG \]

\[ \Rightarrow \angle EGD + \angle FGD = \angle EDG + \angle FDG \]

\[ \Rightarrow \angle EGF = \angle EDF \]

\[ \Rightarrow \angle A = \angle EDF \quad [\because \angle EGF = \angle A] \]

\[ \Rightarrow \angle A = \angle D \]

In \( \Delta ABC \) and \( \Delta DEF \),
\[ AB = DE, \]
\[ AC = DF \, \text{and} \]
\[ \angle A = \angle D \]

\[ \Rightarrow \Delta ABC \cong \Delta DEF \quad \text{(by S.A.S.)} \]

5. **Two right-angled triangles are congruent, if the hypotenuse and one side of one triangle are equal to the hypotenuse and corresponding side of the other triangle.**

**Abbreviated as:** **R.H.S.

The given figure shows two right-angled triangles \( \Delta ABC \) and \( \Delta DEF \) such that:

\[ \angle B = \angle E = 90^\circ; \]

\[ AC = DF \]

and,
\[ AB = DE \]

\[ \therefore \Delta ABC \cong \Delta DEF. \quad \text{[By R.H.S.]} \]

**Proof:**

Let in right angled trianlges \( \Delta ABC \) and \( \Delta DEF \),
\[ \angle B = \angle E = 90^\circ, \, BC = EF \, \text{and} \, AC = DF \]

**Required to prove:** \( \Delta ABC \cong \Delta DEF \)

Produce \( DE \) upto point \( G \) such that \( GE = AB \).

Now show that

\[ \Rightarrow \Delta ABC \cong \Delta GEF \quad \text{(by A.S.A.)} \]

\[ \Rightarrow \angle A = \angle G \, \text{and} \, AC = GF \]

\[ \Rightarrow AC = GF \, \text{and} \, AC = DF \, (\text{given}) \]

\[ \Rightarrow DF = GF \]

\[ \Rightarrow \angle G = \angle D \]

\[ \Rightarrow \angle A = \angle D \, (\text{as,} \, \angle A = \angle G) \]

In \( \Delta ABC \) and \( \Delta DEF \),
\[ \angle A = \angle D \quad \text{(proved above)} \]

\[ \angle B = \angle DEF = 90^\circ \]

\[ \Rightarrow \angle C = \angle DFE \]

In \( \Delta ABC \) and \( \Delta DEF \),
\[ \angle A = \angle D, \, \angle C = \angle DFE \, \text{and} \, AC = DF \]

\[ \Rightarrow \Delta ABC \cong \Delta DEF \quad \text{(by A.S.A.)} \]
CONGRUENCY OF TRIANGLES

Ex.1. Which of the following pairs of triangles are congruent? Give reason.

(a) $\triangle ABC : BC = 4 \text{ cm}, CA = 5 \text{ cm}, \angle C = 70^\circ$
$\triangle PQR : PQ = 4 \text{ cm}, QR = 5 \text{ cm}, \angle Q = 70^\circ$

(b) $\triangle ABC : AB = 4 \text{ cm}, BC = 5 \text{ cm}, \angle B = 70^\circ$
$\triangle PQR : PQ = 4 \text{ cm}, RP = 5 \text{ cm}, \angle R = 70^\circ$

(c) $\triangle ABC : AB = 5 \text{ cm}, BC = 7 \text{ cm}, CA = 9 \text{ cm}$
$\triangle PQR : PQ = 7 \text{ cm}, QR = 5 \text{ cm}, \angle Q = 70^\circ$

Sol. (a) $\triangle ABC : BC = 4 \text{ cm}, CA = 5 \text{ cm}, \angle C = 70^\circ$
$\triangle PQR : PQ = 4 \text{ cm}, QP = 5 \text{ cm}, \angle Q = 70^\circ$

Yes, $\triangle ABC \cong \triangle PQR$

(b) $\triangle ABC : AB = 4 \text{ cm}, BC = 5 \text{ cm}, \angle B = 70^\circ$
$\triangle PQR : PQ = 4 \text{ cm}, RP = 5 \text{ cm}, \angle R = 70^\circ$

$\triangle ABC$ is not congruent to $\triangle PQR$, because two sides are equal but included angle is not same

(c) $\triangle ABC : AB = 5 \text{ cm}, BC = 7 \text{ cm}, CA = 9 \text{ cm}$
$\triangle PQR : PQ = 7 \text{ cm}, QR = 5 \text{ cm}, RP = 9 \text{ cm}$

$\triangle ABC \cong \triangle PQR$

[SAS condition of congruency is satisfied]

Ex.2. $\triangle ABC$ and $\triangle DEF$ are two triangles in which $\angle ABC = 70^\circ, \angle ABC = 50^\circ, \angle DEF = 70^\circ$ and $\angle EDF = 60^\circ$. Prove that the two triangles are congruent.

Sol. In $\triangle ABC$, $\angle BAC = 60^\circ$ and in $\triangle DEF$, $\angle DFE = 50^\circ$

$(\because \text{ Sum of angles of a triangle is } 180^\circ)$

In $\triangle ABC$ and $\triangle DEF$,

$AB = DF$ [Given]
$\angle BAC = \angle EDF = 60^\circ$ [Given]
$\angle ACB = \angle DEF = 70^\circ$ [Given]

Hence, $\triangle ABC \cong \triangle DEF$

[\because AAS-condition of congruency is satisfied]

Ex.3. In the given figure, $\angle R = \angle S$ and $\angle RPQ = \angle PQS$. Prove that $PS = QR$.

Sol. Given: $\angle R = \angle S$ and $\angle RPQ = \angle PQS$

To prove: $PS = QR$

Proof: In $\triangle PQS$ and $\triangle PQR$, we have

$PQ = PQ$
$\angle PSQ = \angle PRQ$
$\angle RPQ = \angle PQS$

Hence, $\triangle PQS \cong \triangle PQR$

$\therefore PS = QR$
Ex.4. In the given figure, \( AB = BC, AD \perp BC, CE \perp AB \). Prove that \( AD = CE \).

Sol. Given: \( AB = BC, AD \perp BC \) and \( CE \perp AB \) [Given]

To prove: \( AD = CE \)

Proof: \( AB = BC \) [Given]

\[ \therefore \angle ACB = \angle CAB \quad \because \text{opposite } \angle s \text{ of equal sides are equal} \]

In \( \triangle ACE \) and \( \triangle ADE \),

\[ \angle ACE = \angle ACD = 90^\circ \]

\[ AC = AC \quad \text{[Common]} \]

\[ \therefore \triangle ACE \cong \triangle ADE \quad \text{[AAS]} \]

\[ \triangle AEC \cong \triangle ADC \]

Hence,

\[ AD = CE \]

Hence proved.

Ex.5. In the given figure, \( AD \) bisects \( \angle A \), \( DE \perp CA \) and \( DF \perp AB \). Prove that \( AF = AE \).

Sol. Given: \( AD \) is the bisector of \( \angle BAC \), \( DE \perp CA \) and \( DF \perp AB \)

To prove: \( AF = AE \)

Proof: In \( \triangle AFD \) and \( \triangle AED \)

\[ AD = AD \quad \text{[Common]} \]

\[ \angle FAD = \angle DAE \quad \because \text{\( AD \) is bisector of \( \angle FAE \)} \]

\[ \angle AFD = \angle AED = 90^\circ \quad \because \text{\( DF \perp AB \) and \( DE \perp AC \)} \]

\[ \therefore \triangle AFD \cong \triangle AED \quad \because \text{\( \triangle AAS\)-conditions of congruency is satisfied} \]

Hence,

\[ AF = AE \]

Ex.6. Two line segments \( PQ \) and \( RS \) bisect each other at \( O \). Prove that:

(a) \( PR = QS \)  
(b) \( \angle RPQ = \angle PQS \)  
(c) \( PS \parallel \parallel RQ \)  
(d) \( PS = RQ \)

Sol. Given: \( PQ \) and \( RS \) bisect each other at \( O \)

To prove: (a) \( PR = QS \)  
(b) \( \angle RPQ = \angle PQS \)  
(c) \( PS \parallel \parallel RQ \)  
(d) \( PS = RQ \)

Proof: (a) In \( \triangle POR \) and \( \triangle QOS \), we have joined \( PR, PS, RQ \) and \( QS, OR, OS \)

\[ PO = OQ \]

\[ \angle POR = \angle QOS \quad \text{[Vertically opposite } \angle s] \]

\[ \therefore \triangle POR \cong \triangle QOS \quad \because \text{\( \triangle SAS\)-condition of congruency is satisfied} \]

Hence,

\[ PR = QS \]

Hence proved.

(b) \[ \therefore \triangle POR \cong \triangle QOS \]

Hence,

\[ \angle RPO = \angle OQS \]

\[ \angle RPQ = \angle PQS \]

Hence proved.

(c) Similarly, \( \triangle QOR \cong \triangle POS \)

\[ \therefore \angle OQR = \angle OPS \]

Hence, \( PS \parallel \parallel QR \)

Hence proved.

(d) \[ \therefore \triangle QOR \cong \triangle POS \]

Hence, \( PS = RQ \)

Hence proved.
Ex. 8. In the given figure, $AB = BC$ and $AD = CD$. Prove that:

(a) $\angle ADE$ is a right angle  
(b) $AE = EC$.

**Sol.**

Given: $AB = BC$ and $AD = CD$

To prove: (a) $\angle ADE = 90^\circ$  
(b) $AE = EC$

**Proof:** In $\triangle ABD$ and $\triangle ACD$, we have

- $AB = BC$  
- $AD = DC$  
- $BD = DB$  

Hence, $\triangle ABD \cong \triangle ACD$  

[Given]  
[Given]  
[Common]  

[SSS-condition of congruency is satisfied]  

[CPCT]

But, $\angle ADB$ and $\angle CDB$ form a linear pair of angles,

- $\angle ADB = \angle CDB = 90^\circ$
- $\angle ADE = \angle CDE = 90^\circ$

Hence proved.

(b) In $\triangle ADE$ and $\triangle CDE$, we have DE is common

- $AD = DC$
- $\angle ADE = \angle EDC = 90^\circ$

So $\triangle ADE \cong \triangle CDE$  

[AS4-condition of congruency is satisfied]  

Hence proved.

Ex. 9. In the given figure, $AB = AC$ and $AP = AQ$. Prove that:

(a) $\triangle APC \cong \triangle AQB$.  
(b) $\triangle BPC \cong \triangle CQB$.

**Sol.**

Given: $AB = AC$ and $AP = AQ$

To prove: (a) $\triangle APC \cong \triangle AQB$  
(b) $\triangle BPC \cong \triangle CQB$

**Proof:**

- $AB - AP = AC - AQ$  
- $PB = QC$  

(i.e.)

(a) In $\triangle APC$ and $\triangle AQB$, we have

- $AP = AQ$
- $\angle PAC = \angle BAQ$
- $AB = AC$

Hence, $\triangle APC \cong \triangle AQB$  

[ASA-condition of congruency is satisfied]  

[CPCT]

(b) In $\triangle BPC$ and $\triangle CQB$, we have $BC$ is common

- $PB = CQ$ and $CP = BQ$  

$\triangle BPC \cong \triangle CQB$  

[SSS-condition of congruency is satisfied]

Hence proved.

Ex. 10. In the given figure, it is given that $AE = AD$ and $BD = CE$. Prove that $\triangle EAB \cong \triangle ADC$.

**Sol.**

Given: $AE = AD$  

and  

$BD = CE$  

In $\triangle EAB$ and $\triangle ADC$,

- $AE = AD$  
- $CE = BD$  

Hence proved.
\[ AE + CE = AD + BD \]
\[ AC = AB \]
Also,
\[ \angle EAB = \angle DAC \]
\[ \triangle AEB \cong \triangle ADC \]

\[ \because \text{SAS condition of congruency is satisfied} \]

Hence proved.

Ex.10. \( ABC \) is a triangle. The bisector of the angle \( BCA \) meets \( AB \) in \( D \). A point \( Y \) lies on \( CD \) such that

\( AD = 4Y \). Prove that \( \angle CAY = \angle ABC \).

Given: \( CD \) is the bisector of \( \angle ACB \), \( AD = AY \)

To prove: \( \angle CAY = \angle ABC \)

Proof: Let \( \angle ACD = \angle BCD = x \)
and
\[ \angle AYD = \angle AYD = y \]
In \( \triangle ADC \), \( x + y + \angle DAC = 180^\circ \)  
[Sum of all \( \angle \)'s of a \( \triangle \) is \( 180^\circ \)]
\[ \angle DAC = 180^\circ - (x + y) \]
Also in \( \triangle ABC \), \( \angle ABC + 2x + 180^\circ - (x + y) = 180^\circ \)
\[ \angle ABC = y - x \]
\[ \angle AYC = 180^\circ - y \]
In \( \triangle AYC \), \( \angle YAC = 180^\circ - [x + (180^\circ - y)] \)
\[ \angle YAC = y - x \]

From equation (i) and (ii), \( \angle CAY = \angle ABC \)

Ex.11. \( P \) is any point in the \( \triangle ABC \) such that the perpendicular drawn from \( P \) on \( AB \) and \( AC \) are equal.

Prove that \( AP \) is the bisector of \( \angle BAC \).

Given: \( P \) is a point in \( \triangle ABC \), \( PQ \perp AB \) and \( PR \perp AC \). Also \( PQ = PR \)

To prove: \( \angle BAP = \angle CAP \)

Proof: In \( \triangle AQP \) and \( \triangle APR \), we have
\[ AP \text{ is common.} \]
\[ PQ = PR \]
\[ \angle AQP = \angle ARP = 90^\circ \]

Hence,
\[ \triangle AQP \cong \triangle APR \]
\[ \therefore \angle QAP = \angle RAP \]
\[ \therefore \triangle AAB \text{ is bisector of } \angle BAC \]

Ex.12. \( ABC \) is a triangle. \( D \) and \( E \) are mid-points of the sides \( AB \) and \( AC \) respectively. \( DE \) is produced to \( F \), such that \( DE = EF \). Prove that \( \triangle ADE \cong \triangle EFC \).

Given: \( D \) and \( E \) are mid-points of \( AB \) and \( AC \) respectively.
\( DE \) is produced to \( F \) such that \( DE = EF \)

To prove: \( \triangle ADE \cong \triangle EFC \)

Proof: In \( \triangle ADE \) and \( \triangle EFC \)
\[ DE = EF \]
\[ \angle ADE = \angle FEC \]
\[ AE = EC \]
\[ \therefore \triangle ADE \cong \triangle EFC \]
\[ \because \text{SAS condition of congruency is satisfied} \]

Hence proved.
Ex. 13.
Sol. In the given figure, $AB = AC$, $BD \perp AC$, $CE \perp AB$. Prove that $BD = CE$.

Given: $BD \perp AC$ and $CE \perp AB$, $AB = AC$

To Prove: $BD = CE$

Proof: In $\triangle ABD$ and $\triangle ACE$, we have

$AB = AC$

$\angle A$ is common

$\angle BDA = \angle AEC = 90^\circ$ [Given]

Hence, $\triangle ABD \cong \triangle ACE$ [AAS-condition of congruency is satisfied]

$BD = EC$ [CPCT]

Hence proved.

Ex. 14.
Sol. Prove that any point on the bisector of an angle is equidistant from the arms of the angle.

Given: $\angle ABC$ and $P$ is any point on bisector of $\angle ABC$.

Const. Draw $PQ \perp AB$ and $PR \perp BC$.

To prove: $PQ = PR$

Proof: In $\triangle BPQ$ and $\triangle BPR$, we have

$BP$ is common.

$\angle QBP = \angle PBR$ [BP is bisector of $\angle ABC$]

$\angle PQB = \angle PRB = 90^\circ$ [Given]

Hence, $\triangle BPQ \cong \triangle BPR$ [AAS-condition of congruency is satisfied]

$PQ = PR$ [CPCT]

Hence proved.

Ex. 15.
Sol. Prove that the internal bisectors of the angles of a triangle are concurrent.

Given: $ABC$ is a triangle. $BI, CI$ internal bisectors of $\angle B$ and $\angle C$ meet at $I$. $AI$ bisects $\angle A$ internally.

To prove: Bisectors of $\angle A$, $\angle B$ and $\angle C$ meet at $I$.

i.e. they are concurrent.

Const. Draw $ID$, $IE$, $IF$ perpendiculars on $BC$, $CA$ and $AB$ respectively.

Proof: In $\triangle IBF$ and $\triangle IBD$,

$\angle BIF = \angle IDB = 90^\circ$ [By construction]

$\angle BID = \angle BIF$ [BI is bisector]

$\therefore \triangle BNF \cong \triangle BMD$ [AAS-condition of congruency is satisfied]

$IF = ID$ [CPCT]

Similarly, in $\triangle IDC$, $IEC$ and in $\triangle IAF$, $IAE$, we can have,

$IE = ID$, $IF = IE$

In $\triangle AIF$ and $\triangle AIE$

$\angle AFI = \angle AEI = 90^\circ$ [By construction]

$AI = AI$ [RHS-condition of congruency is satisfied]

$\therefore \triangle AFI \cong \triangle AEI$ [CPCT]

$\angle FAI = \angle EAI$

$\therefore AI$ bisects $\angle BAC$

Hence, internal bisectors of the angles of a $\triangle$ are concurrent.

Hence proved.
Practice Questions

1. State which of the pair of triangles are congruent. Mention in each case the type of congruency (namely, SSS, SAS, AAS, RHS)

   (a) 
   (b) 
   (c) 
   (d) 

2. In the given fig. (i), prove that \( AE = BD \).

3. In the given fig. (ii), prove that \( AD \) and \( BC \) bisect each other at \( O \).

4. In the given fig. (iii), \( P \) is a point in the interior of \( \angle LMN \) such that \( PA \perp ML \), \( PB \perp MN \) and \( PA = PB \). Show that \( P \) lies on the bisector of \( \angle LMN \).

   Fig. (i)  
   Fig. (ii)  
   Fig. (iii) 

5. In the given fig. (iv), prove that
   (a) \( \triangle ABC \cong \triangle ADC \)  
   (b) \( \angle B = \angle D \)  
   (c) \( AB = CD \) and \( BC = AD \)

6. In the given fig. (v), \( OP \) bisects \( \angle P \) and \( \angle OQP = \angle ORP \), prove that \( \triangle OPQ \cong \triangle OPR \).

7. In the given fig. (vi), \( ABCD \) is a square and \( EF \) is parallel to \( BD \). \( R \) is the mid-point of \( EF \). Prove that
   (a) \( BE = DF \)  
   (b) \( AR \) bisects \( \angle BAD \)  
   (c) If \( AR \) is produced it will pass through \( C \).