ESTD. 2001

LEARN TO LEAD

STEPPING STONE SCHOOL (HIGH)

WORKSHEET – 15 : CLASS – X

Sub – MATHEMATICS. Date – 09.06.20

Topic – Properties of Proportion :—

① Invertendo : If $\frac{a}{b} = \frac{c}{d}$ ⇒ Taking reciprocal of both side we get; $\frac{b}{a} = \frac{d}{c}$, This process is called invertendo

② Alternendo : $a : b = c : d$ ⇒ $\frac{a}{b} = \frac{c}{d}$

⇒ $ad = bc$ ⇒ $\frac{a}{c} = \frac{b}{d}$.

③ Componendo : $\frac{a}{b} = \frac{c}{d}$ ⇒ $\frac{a+b}{b} = \frac{c+d}{d}$

⇒ Proof: $\frac{a}{b} = \frac{c}{d}$ ⇒ $\frac{a}{b} + 1 = \frac{c}{d} + 1$

⇒ $\frac{a+b}{b} = \frac{c+d}{d}$ (Adding '1' on each side)

③ Dividendo : $\frac{a}{b} = \frac{c}{d}$ ⇒ $\frac{a-b}{b} = \frac{c-d}{d}$

Proof: $\frac{a}{b} = \frac{c}{d}$ ⇒ $\frac{a}{b} - 1 = \frac{c}{d} - 1$

⇒ $\frac{a-b}{b} = \frac{c-d}{d}$ (Subtracting '1' from both side)

④ Componendo and Dividendo :— If $\frac{a}{b} = \frac{c}{d}$ given then, $\frac{a+b}{a-b} = \frac{c+d}{c-d}$;

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Proof: From Componendo we get from $\frac{a}{b} = \frac{c}{d}$

$$\Rightarrow \frac{a+b}{b} = \frac{c+d}{d} \quad \text{---- (i)}$$

From Dividendo we get; $\frac{a-b}{b} = \frac{c-d}{d}$ -- (ii)

Dividing (i) and (ii) we get

$$\frac{a+b}{a-b} = \frac{c+d}{c-d}.$$

EXAMPLES: (1) If $\frac{8a-5b}{8c-5d} = \frac{8a+5b}{8c+5d}$; prove that; $\frac{a}{b} = \frac{c}{d}$.

Proof: $\frac{8a-5b}{8c-5d} = \frac{8a+5b}{8c+5d}$

$$\Rightarrow \frac{8a-5b}{8a+5b} = \frac{8c-5d}{8c+5d} \quad \text{(By Alternendo)}$$

Now applying Componendo and dividendo we get

$$\Rightarrow \frac{8a-5b+8a+5b}{8a-5b-8a-5b} = \frac{8c-5d+8c+5d}{8c-5d-8c-5d}$$

$$\Rightarrow \frac{16a}{-10b} = \frac{16c}{-10d} \Rightarrow \frac{a}{b} = \frac{c}{d}$$

(2) If x, y, and z are in continued proportion, prove that; $x^2-y^2 : x^2+y^2 = x-z : x+z$

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Proof: Given x, y, z are continued proportion

So $\frac{x}{y} = \frac{y}{z} \Rightarrow y^2 = zx$

Now, $\frac{x^2 - y^2}{x^2 + y^2} = \frac{x^2 - zx}{x^2 + zx} = \frac{x(x-z)}{x(x+z)} = \frac{x-z}{x+z}$

Ex-3. Using properties of proportion; solve the following equation for 'x' : $\frac{x^3 + 3x}{3x^2 + 1} = \frac{341}{91}$

Ans $\rightarrow$ Applying Componendo and dividendo we get

$$\frac{x^3 + 3x^2 + 3x + 1}{x^3 - 3x^2 + 3x - 1} = \frac{341 + 91}{341 - 91} = \frac{216}{125}$$

$$\Rightarrow \frac{(x+1)^3}{(x-1)^3} = \frac{6^3}{5^3} = \left(\frac{6}{5}\right)^3$$

$\Rightarrow \frac{x+1}{x-1} = \frac{6}{5} \Rightarrow$ Again applying Componendo & dividendo we get

$$\frac{x+1 + x-1}{x+1 - x+1} = \frac{6+5}{6-5} \Rightarrow \frac{2x}{2} = \frac{11}{1} \Rightarrow x = 11.$$

Ex-4: If $a = \frac{4\sqrt{6}}{\sqrt{3} + \sqrt{2}}$; find the value of :

$\frac{a + 2\sqrt{2}}{a - 2\sqrt{2}} + \frac{a + 2\sqrt{3}}{a - 2\sqrt{3}}$. Ans. $\rightarrow$ $a = \frac{(2\sqrt{3})(2\sqrt{2})}{\sqrt{3} + \sqrt{2}}$

$$\Rightarrow \frac{a}{2\sqrt{2}} = \frac{2\sqrt{3}}{\sqrt{3} + \sqrt{2}}$$

Now by applying Componendo and dividendo we get

$$\frac{a+2\sqrt{2}}{a-2\sqrt{2}} = \frac{2\sqrt{3}+\sqrt{3}+\sqrt{2}}{2\sqrt{3}-\sqrt{3}-\sqrt{2}} = \frac{3\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} \quad \text{----- (i)}$$

Again from the relation; $a = \frac{(2\sqrt{2})(2\sqrt{3})}{\sqrt{3}+\sqrt{2}}$, we

write $\frac{a}{2\sqrt{3}} = \frac{2\sqrt{2}}{\sqrt{3}+\sqrt{2}}$; And applying Componendo

and dividendo we get –

$$\frac{a+2\sqrt{3}}{a-2\sqrt{3}} = \frac{2\sqrt{2}+\sqrt{3}+\sqrt{2}}{2\sqrt{2}-\sqrt{3}-\sqrt{2}} = \frac{3\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}} \quad \text{----- (ii)}$$

Now adding (i) and (ii) we get

$$\frac{a+2\sqrt{2}}{a-2\sqrt{2}} + \frac{a+2\sqrt{3}}{a-2\sqrt{3}} = \frac{3\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} + \frac{3\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}}$$

$$= \frac{3\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} - \frac{3\sqrt{2}+\sqrt{3}}{\sqrt{3}-\sqrt{2}}$$

$$= \frac{3\sqrt{3}+\sqrt{2}-3\sqrt{2}-\sqrt{3}}{\sqrt{3}-\sqrt{2}} = \frac{2\sqrt{3}-2\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$

$$= \frac{2(\sqrt{3}-\sqrt{2})}{(\sqrt{3}-\sqrt{2})} = 2 \text{ (Ans).}$$

Hence $\frac{a+2\sqrt{2}}{a-2\sqrt{2}} + \frac{a+2\sqrt{3}}{a-2\sqrt{3}} = 2$. (Ans)

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EXERCISE:

Q1) If $p = \frac{4xy}{x+y}$; find the value of

$$\frac{p+2x}{p-2x} + \frac{p+2y}{p-2y}.$$

Q2) If a, b, c are in Continued proportion; prove that i) $\frac{a^2+ab+b^2}{b^2+bc+c^2} = \frac{a}{c}$

ii) $\frac{a^2+b^2+c^2}{(a+b+c)^2} = \frac{a-b+c}{a+b+c}.$

Q3) Using properties of proportion, solve for 'x'

$$\frac{\sqrt{x+1}+\sqrt{x-1}}{\sqrt{x+1}-\sqrt{x-1}} = \frac{4x-1}{2}.$$

Q4) If $\frac{\sqrt{a+3b}+\sqrt{a-3b}}{\sqrt{a+3b}-\sqrt{a-3b}} = x$; then prove that $3bx^2 - 2ax + 3b = 0$

Q5) If $\frac{x^3+3xy^2}{3x^2y+y^3} = \frac{m^3+3mn^2}{3m^2n+n^3}$; show that $nx = my$.

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