



STEPPING STONE SCHOOL (HIGH)

WORKSHEET - 14 :

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Sub - MATHEMATICS : Date : 08.06.2020
Topic - RATIO & PROPORTION :

PROPORTION : Four non-zero quantities a, b, c, d are said to be in proportion if

$$a : b = c : d .$$

Continued proportion : Three non zero quantities a, b, c are in continued proportion

$$\text{if } a : b = b : c \Rightarrow \frac{a}{b} = \frac{b}{c} \Rightarrow b^2 = ac$$

$\Rightarrow b = \pm \sqrt{ac}$; 'b' is called the mean proportion between 'a' and 'c' ; 'c' is called third proportion.

Example : \Rightarrow Find the mean-proportional between 6.25 and 0.16 .

Ans \rightarrow If the third proportional be 'x' then

$$x^2 = 6.25 \times 0.16 = \frac{625 \times 16}{10000} = \frac{(25 \times 4)(25 \times 4)}{100 \times 100}$$

$$\Rightarrow x^2 = \frac{100 \times 100}{100 \times 100} = 1 \Rightarrow x = \pm 1$$

Ex-2 \rightarrow What number should be subtracted from each of the numbers 23, 30, 57 and 78 ; so that the remainders are in proportion ?

Ans \rightarrow Let the number subtracted be 'x'

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$$\therefore (23-x) : (30-x) = (57-x) : (78-x)$$

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$$\Rightarrow \frac{23-x}{30-x} = \frac{57-x}{78-x} \Rightarrow$$

$$1794 - 101x + x^2 = 1710 - 87x + x^2$$

$$\Rightarrow 14x = 84 \Rightarrow x = 6.$$

Ex-3: Given '6' is the mean proportional between 'x' and 'y' and '48' is the third proportional to 'x' and 'y'. Find 'x' and 'y'.

Ans \rightarrow As '6' is the mean proportion between 'x' and 'y', hence $xy = 6^2 = 36$ — (i)

And '48' is the third proportion between 'x' and 'y'.

$$\text{So } x : y = y : 48 \Rightarrow y^2 = 48x \text{ — (ii)}$$

From (i) and (ii) we get: —

$$y^2 = 48 \cdot \left(\frac{36}{y}\right) \left[\because x = \frac{36}{y} \right]$$

$$\Rightarrow y^3 = 4 \times 12 \times 12 \times 3 = 12 \times 12 \times 12 = 12^3$$

$$\Rightarrow y = 12 \quad \text{And } x = \frac{36}{12} = 3.$$

Ex-4: If 'y' is mean proportional between 'x' and 'z', then prove that \rightarrow

$$\frac{x^r - y^r + z^r}{x^{-2} - y^{-2} + z^{-2}} = y^4.$$

Ans \rightarrow As given, $y^r = zx$; (y is mean proportional betn z and x)

$$\text{So LHS} = \frac{x^r - y^r + z^r}{\frac{1}{x^r} - \frac{1}{zx} + \frac{1}{z^r}} = \frac{x^r - y^r + z^r}{\frac{z^r - zx + x^r}{z^r x^r}}$$

$$\Rightarrow \frac{(x^r - y^r + z^r) z^r x^r}{(x^r + z^r - y^r) z^r x^r} \quad [\because zx = y^r]$$

$$= \frac{z^r x^r}{z^r x^r} = (y^r)^r = y^4 = \text{RHS.}$$

EXERCISES:

① Find the third proportional to $\frac{x}{y} + \frac{y}{x}$ and $\sqrt{x^r + y^r}$.

Q2) If a, b, c are in continued proportion, show that, $\frac{a^r + b^r}{b(a+c)} = \frac{b}{b^r + c^r}$

③ If a, b, c, d are in proportion; then prove that:

$$\sqrt{\frac{4a^r + 9b^r}{4c^r + 9d^r}} = \left[\frac{xa^3 - 5yb^3}{xc^3 - 5yd^3} \right]^{\frac{1}{3}}$$

④ If $p+r = mq$ and $\frac{1}{q} + \frac{1}{s} = \frac{m}{r}$, then prove that: $p:s = q:r$.

P.T.O \rightarrow

⑤ Find two numbers such that the mean proportional between them is 12 and the third proportional to them is 96

⑥ If $p:q = r:s$; then show that,
 $mp + nq : d = mr + ns : s$

⑦ If three quantities are in continued proportion. Show that the ratio of the first to the third is the duplicate ratio of the first to the second.

⑧ If 'q' is the mean proportional between 'p' and 'r', then prove that

$$p^n - q^n + r^n = q^n \left(\frac{1}{p^n} - \frac{1}{q^n} + \frac{1}{r^n} \right)$$

→ ENDC ←