ESTD. 2001

LEARN TO LEAD

STEPPING STONE SCHOOL (HIGH)

CLASS: IX

SUBJECT - MATH

TOPIC : STATISTICS

Dated : 09.06.2020

WORKSHEET # 15

INTRODUCTION

The word 'Statistics' seems to have derived from the Latin word 'status' which means a 'political state'. Originally, statistics was simply the collection of numerical data on some aspects of life of the people useful to the government. However, with the passage of time, its scope broadened. Today, statistics means collection of facts or information concerning almost every aspect of life of the people with a definite purpose in the form of numerical data, organisation, summarisation and presentation of data by tables and graphs (charts), analysing the data and drawing inferences (meaningful predictions) from the data.

20.1 SOME TERMS RELATED TO STATISTICS

- **Primary data.** *The information collected by the investigator himself or herself with a definite purpose in his or her mind is called* ***primary data.***
- **Secondary data.** *The information gathered from a source which already had the information stored is called* ***secondary data.***
- **Raw data.** *The numerical data recorded in its original form as it is collected by the investigator or received from some source is called* ***raw data.***
- **Variable.** *A quantity which is being measured in an experiment (or survey) is called a* ***variable.*** Height, age and weight of people, income and expenditure of people, number of members in a family, number of workers in a factory, marks obtained by students in a test, the number of runs scored in a cricket match etc., are examples of variables.

  Variables are of two types:

  (*i*) **Continuous variable.** *A variable which can take any value between two given values is called a* ***continuous variable.***

  For example, height, age and weight of people are continuous variables.

  (*ii*) **Discontinuous (discrete) variable.** *A variable which cannot take all possible values between two given values is called a* ***discontinuous or discrete variable.***

  For example, the number of members in a family and the number of workers in a factory are discrete variables (since the variable cannot take any value between 1 and 2, 2 and 3 etc.)
- **Range.** *The difference between the maximum and minimum values of a variable is called its* ***range.***
- **Variate.** *A particular value of a variable is called* ***variate (observation).***
- **Frequency.** *The number of times a variate (observation) occurs in a given data is called frequency of that variate.*
- **Frequency distribution.** *A tabular arrangement of given numerical data showing the frequency of different variates is called* ***frequency distribution,*** *and the table itself is called* ***frequency distribution table.***

20.1.1 Tabulation of raw data

Suppose there are 32 students in class IX in a school and in an examination, out of 50 marks, the marks scored by them are as follows :

39, 44, 25, 11, 21, 25, 44, 25, 7, 40, 43, 44, 49, 14, 11, 14, 25, 28, 28, 39, 44, 37, 21, 40, 43, 3, 37, 25, 25, 21, 37, 28.

The data in this form is the **raw** (or **ungrouped** or **unclassified) data**. Here, the number of marks obtained is the **variable** and each entry in the above list is an **observation** (or **variate**).

Suppose we wish to analyse the achievement of the students in the examination, the data in the above form does not give much information.

Let us arrange the above data in ascending or descending order. The above data in the ascending order is :

3, 7, 11, 11, 14, 14, 21, 21, 21, 25, 25, 25, 25, 25, 25, 28, 28, 28, 37, 37, 37, 39, 39, 40, 40, 43, 43, 44, 44, 44, 44, 49.

The presentation of data in this form gives better information. The data arranged in this form is called **arrayed data**. However, the presentation of data in this form is quite tedious and time-consuming, particularly when the number of observations is large.

To make it easily understandable, we present the above data in the form of a table called ***frequency distribution table*** (*for raw data*). To prepare table, we take each observation from the data, one at a time, and mark a stroke (I) called **tally mark** in the next column opposite to the variate. For convenience, we write tally marks in bunches of five, the fifth one crossing the four diagonally. The number of *tally marks* opposite to a variate is its **frequency** and it is written in the next column opposite to tally marks of the variate. Note that the sum of all the frequencies is equal to the total number of observations in the given data.

The frequency distribution table for the above raw (ungrouped) data is given below:

Marks obtained	*Tally marks*	*Frequency*
3	I	1
7	I	1
11	II	2
14	II	2
21	III	3
25	~~IIII~~ I	6
28	III	3
37	III	3
39	II	2
40	II	2
43	II	2
44	IIII	4
49	I	1
Total		32

The above table is called *simple (or ungrouped) frequency distribution table.*

20.2 MEAN AND MEDIAN OF UNGROUPED DATA

20.2.1 Mean of ungrouped data

Mean (or ***arithmetic average***) *of a number of observations is the sum of the values of all the observations divided by the total number of observations.*

The mean of n observations (variates) $x_1, x_2, x_3, \ldots, x_n$ *is given by the formula:*

$$\text{Mean} = \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n} = \frac{\Sigma x_i}{n}$$

where $\Sigma x_i = x_1 + x_2 + x_3 + \ldots + x_n$.

Thus, $\text{Mean} = \dfrac{\text{sum of all observations}}{\text{total number of observations}}$.

Note The Greek letter Σ (read as sigma) represents the sum.

Illustrative Examples

Example 1 The following are the ages (in years) of 10 teachers in a school :

32, 28, 54, 41, 38, 40, 23, 33, 26, 35.

Find the mean age of these teachers.

Solution. The sum of the ages (in years) of all the 10 teachers

$= 32 + 28 + 54 + 41 + 38 + 40 + 23 + 33 + 26 + 35$

$= 350.$

$\therefore$ Mean age $= \dfrac{\text{sum of ages of all the teachers}}{\text{total number of teachers}}$

$= \dfrac{350}{10}$ years = 35 years.

Example 2 The marks obtained (out of 25) by 15 students in a monthly test are :

11, 09, 07, 03, 18, 21, 13, 15, 18, 04, 06, 17, 22, 13, 15.

(*i*) Find the mean of their marks.

(*ii*) Find the mean of their marks when the marks of each student are increased by 2.

Solution. (*i*) The sum of the marks of all the 15 students

$= 11 + 09 + 07 + 03 + 18 + 21 + 13 + 15 + 18 + 04 + 06 + 17 + 22 + 13 + 15$

$= 192.$

$\therefore$ Mean of marks $= \dfrac{\text{sum of marks of all the students}}{\text{total number of students}}$

$= \dfrac{192}{15} = \dfrac{64}{5} = 12{\cdot}8$

(*ii*) When the marks of each students are increased by 2, then the sum of their marks increases by 15 × 2 *i.e.* by 30.

$\therefore$ The new sum of marks of all the students = 192 + 30 = 222.

$\therefore$ The new mean of their marks $= \dfrac{\text{new sum of marks}}{\text{number of students}}$

$= \dfrac{222}{15} = \dfrac{74}{5} = 14{\cdot}8$

Note that the new mean of marks also increases by 2.

Example 3 For the given data: 11, 15, 17, $y + 1$, 19, $y - 2$, 3; if the mean is 14, then find the value of y.

Solution. Mean $= \dfrac{\text{sum of the given numbers}}{\text{number of observations}}$

$$\Rightarrow \quad \frac{11 + 15 + 17 + y + 1 + 19 + y - 2 + 3}{7} = 14 \text{ (given)}$$

$$\Rightarrow \quad 64 + 2y = 98 \Rightarrow 2y = 34 \Rightarrow y = 17.$$

Hence, the value of y is 17.

Example 4 Find the mean of first seven prime numbers.

Solution. First seven prime numbers are 2, 3, 5, 7, 11, 13, 17.

Sum of these prime numbers $= 2 + 3 + 5 + 7 + 11 + 13 + 17 = 58.$

$$\therefore \quad \text{Their mean} = \frac{\text{sum of numbers}}{\text{number of numbers}} = \frac{58}{7} = 8\frac{2}{7}.$$

Example 5 Find the mean of the (positive) factors of 24.

Solution. The (positive) factors of 24 are : 1, 2, 3, 4, 6, 8, 12, 24.

Number of these factors = 8 and

the sum of these factors $= 1 + 2 + 3 + 4 + 6 + 8 + 12 + 24 = 60.$

$$\therefore \quad \text{Mean} = \frac{\text{sum of factors}}{\text{no. of factors}} = \frac{60}{8} = \frac{15}{2} = 7{\cdot}5$$

Example 6 The mean of 6 observations is 17·5. If five of them are 14, 9, 23, 25 and 10, find the sixth observation.

Solution. Let the sixth observation be x. By def.,

$$\therefore \quad \text{mean} = \frac{\text{sum of all the observations}}{\text{number of observations}}$$

$$\Rightarrow \quad 17{\cdot}5 = \frac{14 + 9 + 23 + 25 + 10 + x}{6}$$

$$\Rightarrow \quad 17{\cdot}5 = \frac{81 + x}{6}$$

$$\Rightarrow \quad 81 + x = 17{\cdot}5 \times 6$$

$$\Rightarrow \quad x = 105 - 81 = 24.$$

Hence, the sixth observation is 24.

Example 7 The following are the weights (in kg) of 8 students of a class :

50, 44·5, 48·7, 45·1, 50·4, 43, 51, 49·3.

(i) Find the mean weight.

(ii) If a teacher, whose weight is 62 kg, is also included then what will be the mean weight?

Solution. (i) The sum of weights (in kg) of 8 students

$= 50 + 44{\cdot}5 + 48{\cdot}7 + 45{\cdot}1 + 50{\cdot}4 + 43 + 51 + 49{\cdot}3$

$= 382.$

$$\therefore \quad \text{Mean weight} = \frac{\text{sum of weights of all students}}{\text{number of students}}$$

$$= \frac{382}{8} \text{ kg} = 47{\cdot}75 \text{ kg}.$$

(ii) If the teacher, whose weight is 62 kg, is included then the sum of weights of all observations = sum of weights of students + weight of teacher

= 382 kg + 62 kg = 444 kg.

Number of observations = 8 + 1 = 9.

∴ Mean weight = $\frac{444}{9}$ kg = $\frac{148}{3}$ kg = $49\frac{1}{3}$ kg.

Example 8 The mean of 10 numbers is 55. If one number is included, their mean becomes 60. Find the included number.

Solution. The mean of 10 numbers is 55

⇒ the sum of 10 numbers = 55 × 10 = 550.

When one number is included, the mean becomes 60

i.e. the mean of 11 numbers is 60

⇒ sum of 11 numbers = 60 × 11 = 660.

∴ The included number = sum of 11 numbers − sum of 10 numbers

= 660 − 550 = 110.

Example 9 The mean height of 10 students is 151·8 cm. Two more students of heights 157·6 cm and 154·4 cm join the group. What is the new mean height?

Solution. Mean height of 10 students = $\frac{\text{sum of heights of 10 students}}{10}$

∴ 151·8 cm = $\frac{\text{sum of heights of 10 students}}{10}$

⇒ sum of heights of 10 students = (151·8 × 10) cm = 1518 cm.

Now two more students of heights 157·6 cm and 154·4 cm join the group.

∴ Sum of heights of 12 students = (1518 + 157·6 + 154·4) cm = 1830 cm.

∴ New mean height = $\frac{\text{sum of heights of 12 students}}{12}$

= $\frac{1830}{12}$ cm = 152·5 cm.

Example 10 There are 50 numbers. If each number is subtracted from 53, then the mean of the numbers so obtained is − 3·5. Find the mean of the given numbers.

Solution. Let the given 50 numbers be $x_1, x_2, x_3, \ldots, x_{50}$.

When each number is subtracted from 53, the numbers so obtained are $53 - x_1$, $53 - x_2$, $53 - x_3, \ldots, 53 - x_{50}$.

Mean of these numbers = $\frac{\sum_{i=1}^{50}(53 - x_i)}{50}$ = −3·5 (given)

⇒ $53 \times 50 - \sum_{i=1}^{50} x_i = -3.5 \times 50$

⇒ $53 \times 50 + 3.5 \times 50 = \sum_{i=1}^{50} x_i$

⇒ $56.5 \times 50 = \sum_{i=1}^{50} x_i$

⇒ $\frac{\sum_{i=1}^{50} x_i}{50} = 56.5$ ⇒ mean of given numbers = 56·5

Example 11 The mean marks (out of 100) of boys and girls in an examination are 70 and

73 respectively. If the mean marks of all the students in that examination are 71, find the ratio of number of boys to the number of girls.

Solution. Let the number of boys be x and that of girls be y.

As the mean of marks scored by boys is 70,

$\therefore$ the sum of marks scored by x boys = $70x$.

Also the mean of marks scored by girls is 73,

$\therefore$ the sum of marks scored by y girls = $73y$.

$\therefore$ the sum of marks scored by all $(x + y)$ students = $70x + 73y$.

Since the mean of marks scored by all students is 71,

$$\therefore \quad \frac{70x + 73y}{x + y} = 71 \Rightarrow 70x + 73y = 71x + 71y$$

$$\Rightarrow \quad 2y = x \Rightarrow \frac{x}{y} = \frac{2}{1}.$$

Hence, the ratio of number of boys to that of girls = 2 : 1.

Example 12 Mean temperature of a city of a certain week was 25°C. If the mean temperature of Monday, Tuesday, Wednesday and Thursday was 23°C and that of Thursday, Friday, Saturday and Sunday was 28°, find the temperature of Thursday.

Solution. Mean temperature of the week = 25°C.

$\therefore$ The sum of temperatures of 7 days of the week = 7 × 25°C = 175°C ...(i)

Sum of temperatures of Monday, Tuesday, Wednesday and Thursday

= 4 × 23°C = 92°C ...(ii)

Sum of temperatures of Thursday, Friday, Saturday and Sunday

= 4 × 28°C = 112°C ...(iii)

$\therefore$ Sum of temperatures of Monday to Sunday and Thursday

= 92°C + 112°C [using (ii) and (iii)]

= 204°C ...(iv)

$\therefore$ Temperature of Thursday = 204°C − 175°C [using (iv) and (i)]

= 29°C.

Example 13 In an examination, the mean of marks scored by a class of 40 students was calculated as 72·5. Later on, it was detected that the marks of one student were wrongly copied as 48 instead of 84. Find the correct mean.

Solution. Mean of marks = $\frac{\text{Incorrect sum of marks of 40 students}}{40}$

$$\therefore \quad 72{\cdot}5 = \frac{\text{Incorrect sum of marks of 40 students}}{40}$$

$\Rightarrow$ Incorrect sum of marks of 40 students = 72·5 × 40 = 2900.

Since the marks of one student were wrongly copied as 48 instead of 84,

correct sum of marks of 40 students = 2900 − 48 + 84 = 2936.

$$\therefore \quad \text{Correct mean} = \frac{2936}{40} = 73{\cdot}4$$

20.2.2 Median of ungrouped data

Median is the central value (or middle observation) of a statistical data if it is arranged in ascending or descending order.

Thus, if there are n observations (variates) $x_1, x_2, x_3, \ldots, x_n$ arranged in ascending or descending order, then

$$\text{Median} = \begin{cases} \dfrac{n+1}{2} \text{ th observation, if } n \text{ is odd} \\ \dfrac{\dfrac{n}{2} \text{ th observation} + \left(\dfrac{n}{2}+1\right) \text{ th observation}}{2}, \text{ if } n \text{ is even.} \end{cases}$$

Illustrative Examples

Example 1 Find the median of the following data:

5, 3, 12, 0, 7, 11, 4, 3, 8.

Solution. Arranging the given data in ascending order, we get

0, 3, 3, 4, 5, 7, 8, 11, 12.

Total number of observations = n = 9, which is odd.

$$\therefore \quad \text{Median} = \frac{n+1}{2} \text{ th observation}$$

$$= \frac{9+1}{2} \text{ th observation}$$

= 5th observation, which is 5.

Hence, median = 5.

Example 2 The number of goals scored by a football team in a series of matches are:

3, 1, 0, 7, 5, 3, 3, 4, 1, 2, 0, 2.

Find the median of this data.

Solution. Arranging the number of goals scored by the team in ascending order, we get

0, 0, 1, 1, 2, 2, 3, 3, 3, 4, 5, 7.

Total number of observations (matches played) = n = 12, which is even.

$$\therefore \quad \text{Median} = \frac{\frac{n}{2} \text{ th observation} + \left(\frac{n}{2}+1\right) \text{ th observation}}{2}$$

$$= \frac{\text{6th observation} + \text{7th observation}}{2} = \frac{2+3}{2} = \frac{5}{2} = 2{\cdot}5$$

Example 3 If the numbers 3, 6, 7, 10, x, 15, 19, 20, 25, 28 are in ascending order and their median is 13, calculate the value of x.

Solution. The numbers 3, 6, 7, 10, x, 15, 19, 20, 25, 28 are in ascending order.

Total number of observations = n = 10, which is even.

$$\therefore \quad \text{Median} = \frac{\frac{n}{2} \text{ th observation} + \left(\frac{n}{2}+1\right) \text{ th observation}}{2}$$

$$= \frac{\text{5th observation} + \text{6th observation}}{2} = \frac{x+15}{2}.$$

According to given, $13 = \dfrac{x+15}{2}$

$\Rightarrow \quad x + 15 = 26 \Rightarrow x = 11.$

Hence, the value of x is 11.

EXERCISE

1. Find the mean of 8, 6, 10, 12, 1, 3, 4, 4.
2. 5 people were asked about the time in a week they spend in doing social work in their community. They replied 10, 7, 13, 20 and 15 hours, respectively. Find the mean time in a week devoted by them for social work.
3. The enrolment of a school during six consecutive years was as follows:

   1620, 2060, 2540, 3250, 3500, 3710.

   Find the mean enrolment.
4. Find the mean of the first twelve natural numbers.
5. (i) Find the mean of the first six prime numbers.

   (ii) Find the mean of the first seven odd prime numbers.
6. (i) The marks (out of 100) obtained by a group of students in a Mathematics test are 81, 72, 90, 90, 85, 86, 70, 93 and 71. Find the mean marks obtained by the group of students.

   (ii) The mean of the age of three students Vijay, Rahul and Rakhi is 15 years. If their ages are in the ratio 4 : 5 : 6 respectively, then find their ages.
7. The mean of 5 numbers is 20. If one number is excluded, mean of the remaining numbers becomes 23. Find the excluded number.
8. The mean of 25 observations is 27. If one observation is included, the mean still remains 27. Find the included observation.
9. The mean of 5 observations is 15. If the mean of first three observations is 14 and that of the last three is 17, find the third observation.
10. The mean of 8 variates is 10·5. If seven of them are 3, 15, 7, 19, 2, 17 and 8, then find the 8th variate.
11. The mean weight of 8 students is 45·5 kg. Two more students having weights 41·7 kg and 53·3 kg join the group. What is the new mean weight?
12. Mean of 9 observations was found to be 35. Later on, it was detected that an observation 81 was misread as 18. Find the correct mean of the observations.
13. A student scored the following marks in 11 questions of a question paper :

    7, 3, 4, 1, 5, 8, 2, 2, 5, 7, 6.

    Find the median marks.
14. Calculate the mean and the median of the numbers :

    2, 3, 4, 3, 0, 5, 1, 1, 3, 2.
15. A group of students was given a special test in Mathematics. The test was completed by the various students in the following time in (minutes) :

    24, 30, 28, 17, 22, 36, 30, 19, 32, 18, 20, 24.

    Find the mean time and median time taken by the students to complete the test.
16. In a Science test given to a group of students, the marks scored by them (out of 100) are:

    41, 39, 52, 48, 54, 62, 46, 52, 40, 96, 42, 40, 98, 60, 52.

    Find the mean and median of this data.
17. The points scored by a Kabaddi team in a series of matches are as follows :

    7, 17, 2, 5, 27, 15, 8, 14, 10, 48, 10, 7, 24, 8, 28, 18.

    Find the mean and the median of the points scored by the Kabaddi team.