ESTD. 2001

LEARN TO LEAD

STEPPING STONE SCHOOL (HIGH)

Sub- MATHEMATICS: CLASS-X

Worksheet No-23. Date-26.06.20

Topic: STANDARD PROBLEMS ON G.P & A.P. Chapt.-11

Example 1. If $a, b,$ and c are in A.P; a, x, b are in G.P where b, y, c are also in G.P. Show that; x^2, b^2, y^2 are in A.P

Ans: As a, b, c are in A.P; Hence, $2b = a+c$ ---(i)

And a, x, b in GP $\Rightarrow x^2 = ab$

" b, y, c in G.P $\Rightarrow y^2 = bc$

So $x^2 + y^2 = ab + bc = b(a+c) = b \cdot 2b = 2b^2$

$\Rightarrow x^2 + y^2 = 2b^2 \Rightarrow x^2, b^2, y^2$ are in A.P.

Ex-2: If a, b, c are in G.P and a, x, b, y, c are in A.P. Prove that; i) $\frac{1}{x} + \frac{1}{y} = \frac{2}{b}$

ii) $\frac{a}{x} + \frac{c}{y} = 2$

Ans: Given, $b^2 = ac$, $2x = a+b$ and $2y = b+c$

So $\frac{1}{x} + \frac{1}{y} = \frac{2}{a+b} + \frac{2}{b+c} = 2\left[\frac{1}{a+b} + \frac{1}{b+c}\right]$ (Using above relation)

$= 2\left[\frac{2b+a+c}{(b+a)(b+c)}\right] = \frac{2[2b+a+c]}{b^2+b(a+c)+ac}$ P.T.O →

$$= \frac{2(2b+a+c)}{b^2+b(a+c)+b^2} \quad (\because b^2 = ac)$$

$$= \frac{2(2b+a+c)}{2b^2+b(a+c)} = \frac{2(2b+a+c)}{b(2b+a+c)} = \frac{2}{b}.$$

ii) $\frac{a}{x} + \frac{c}{y} = 2\left[\frac{a}{a+b} + \frac{c}{b+c}\right]$ (Putting 'x' and 'y' in 'a, b, c)

$$= 2\left[\frac{ab+ac+ac+bc}{b^2+b(a+c)+ac}\right] \quad \text{Put; } ac = b^2$$

$$= 2\left[\frac{ab+bc+2ac}{b^2+b(a+c)+b^2}\right] = \frac{2(ab+bc+2b^2)}{ab+ac+2b^2}$$

$= 2.$ Proved.

Ex-3: If the sum of $1+2+2^2+ \text{------} + 2^{n-1}$ is 255; find the value of 'n'

Ans → Here $S_n = 1+2+2^2+ \text{---} 2^{n-1}$; has 'n' terms and first term is '1'; so by formula

$S_n = \frac{a(r^n-1)}{r-1}$; $r =$ Common difference $= 2$ (here)

$$= \frac{1.(2^n-1)}{2-1} = 2^n - 1 = 255, \text{ Given.}$$

So $2^n = 256 = 2^8 \Rightarrow 2^n = 2^8 \Rightarrow n = 8.$

P.T.O

Ex-4: Find a G.P for which the sum of first two terms is '-4' and the fifth term is 4 times the third term.

Ans – Let the first term and Common ratio be 'a' and 'r' respectively. Hence; $a + ar = -4$ --- (i)

and $ar^4 = 4ar^2$ — (ii)

$\Rightarrow r^2 = 2^2 \Rightarrow r = \pm 2$. So from (i)

we get $a(1+r) = -4 \Rightarrow a(1+2) = -4$ if $r = +2$

So $a = -4/3$. So the series becomes –

$-\frac{4}{3}, -\frac{8}{3}, -\frac{16}{3}$ ------ . Now if $r = -2$ then

$a(1-2) = -4 \Rightarrow a = 4$, Hence the series become

$4, -8, 16, -32, 64$ ------ .

Exercises: (1) If a, b, c are in G.P and $a^{\frac{1}{x}} = b^{\frac{1}{y}} = c^{\frac{1}{z}}$; prove that, x, y, z are in A.P

(2) If $(p+q)^{th}$ term of a G.P is 'm' and $(p-q)^{th}$ term is 'n'; Show that pth term is $\sqrt{mn}$ and qth term is $m\left(\frac{n}{m}\right)^{\frac{p}{2q}}$.

P.T.O →

Ex3. If a, b, c are in A.P ; b, c, d are in G.P and $\frac{1}{c}, \frac{1}{d}, \frac{1}{e}$ are in A.P ; Prove that ; a, c, e are in G.P.

[Hints: $b = \frac{a+c}{2}$ —(i) $c^2 = bd$ --(ii) and

$\frac{2}{d} = \frac{1}{c} + \frac{1}{e} \Rightarrow d = \frac{2ce}{a+c}$ —(iii). Substitute the values of 'b' and 'd' from (i) and (iii) in (ii)]

Ex4) If pth, qth and rth term of an A.P are in G.P ; Prove that the Common ratio of the G.P is $\frac{q-r}{p-q}$. [Apply the property: If $\frac{a}{b} = \frac{c}{d}$, then each ratio is $\frac{a-c}{b-d}$ ie

$\frac{t_q}{t_p} = \frac{t_r}{t_q} \Rightarrow C.r = \frac{t_q - t_r}{t_p - t_q}$]

Ex-5) The lengths of three unequal edges of rectangular solid block are in G.P. The volume of the block is 216 cm^3 and its surface area is 252 cm^2. Find the length of the longest edge.

Ex-6) If a, b, c are in A.P as well as in G.P also. Prove that, $a = b = c$

Home work $\longrightarrow$ 11C and 11D Problems.

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