ESTD. 2001

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STEPPING STONE SCHOOL (HIGH)

Sub:- Mathematics : Class - X.

Worksheet - 22. Date: 25.06.20

Topic - G.P : Chapter - 11.

More Problems:

Example 1. If the sum of three terms in G.P is 26 and the product of them is 216, then find the terms.

Ans → Let the three numbers in G.P be $\frac{a}{r}$, a and ar. Hence $\frac{a}{r} \cdot a \cdot ar = a^3 = 216$

$a^3 = 6^3 \Rightarrow a = 6.$

As the sum of the terms = 26

So, $\frac{a}{r} + a + ar = 26$

$\Rightarrow a\left(\frac{r^2 + r + 1}{r}\right) = 26 \Rightarrow 6(r^2 + r + 1) = 26r$ [$\because a = 6$]

$\Rightarrow 3r^2 + 3r + 3 = 13r$

$\Rightarrow 3r^2 - 10r + 3 = 0 \Rightarrow 3r^2 - 9r - r + 3 = 0$

$\Rightarrow (3r - 1)(r - 3) = 0 \Rightarrow r = 3, \frac{1}{3}$

So the terms are $\frac{6}{3}$, 6, 6×3 ie 2, 6, 18 or 18, 6, 2 (if $r = \frac{1}{3}$).

P.T.O

Example 2. The fifth, eight and eleventh term of a G.P are 'p' 'q' and 'r'. Show that; $q^2 = pr$.

Ans → Given $t_5 = p \Rightarrow ar^4 = p$

$t_8 = q \Rightarrow ar^7 = q$

$t_{11} = r \Rightarrow ar^{10} = r$

where 'a' is the first term and 'r' is the Common ratio

Now $pr = a^2r^{14}$ and $q^2 = (ar^7)^2 = a^2r^{14}$

Hence $pr = a^2r^{14} = q^2 \Rightarrow q^2 = pr$.

Example 3. The first and nth term of a G.P are 'a' and 'b' respectively, and if 'P' is the product of first 'n' terms, prove that, $P^2 = (ab)^n$

Ans → As $t_n = b \Rightarrow b = ar^{n-1} \Rightarrow r^{n-1} = \frac{b}{a}$ ---(i)

So P = product of first 'n' terms

$= a \cdot ar \cdot ar^2 \cdot ar^3 \text{----} ar^{n-1}$

$= a^n \cdot r^{1+2+3+\cdots+(n-1)} = a^n \cdot r^{\frac{(n-1)n}{2}}$

[As $1+2+3+\cdots+n = \frac{n(n+1)}{2}$] $= a^n\left(r^{n-1}\right)^{\frac{n}{2}}$

P.T.O →

So $P = a^n \left(\frac{b}{a}\right)^{\frac{n}{2}} \Rightarrow P = a^{\frac{n}{2}} \cdot b^{n/2}$

$\Rightarrow P^2 = (ab)^n$. (Proved).

Example 4: If for a G.P, p^{th}, q^{th} and r^{th} terms are 'a', 'b' and 'c' respectively; prove that

$(q-r)\log a + (r-p)\log b + (p-q)\log c = 0$

Ans- $t_p = a = x \cdot y^{p-1}$ if 'x' is the first term and 'y' is the common ratio

Similarly $t_q = b = xy^{q-1}$ and $t_r = x \cdot y^{r-1}$

Now; $a^{q-r} \cdot b^{r-p} \cdot c^{p-q}$

$= (xy^{p-1})^{q-r} (xy^{q-1})^{r-p} (x \cdot y^{r-1})^{p-q}$

$= x^{q-r+r-p+p-q} \cdot y^{(p-1)(q-r)+(q-1)(r-p)+(r-1)(p-q)}$

$= x^0 \cdot y^{pq-pr+qr-pq+pr-qr-(q-r+r-p+p-q)}$

$= y^{0-0} = y^0 = 1$

Hence: $a^{q-r} \cdot b^{r-p} \cdot c^{p-q} = 1$

Taking 'log' on both sides we get

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$$\log a^{p-q} \cdot b^{r-p} \cdot c^{p-q} = \log 1 = 0$$

$$\Rightarrow (p-q)\log a + (r-p)\log b + (p-q)\log c = 0 \text{ ; proved.}$$

Exercises:

① Find three numbers in G.P whose product is 216, and the sum of their products in pairs is 156.

② Three numbers whose sum is 21, are in A.P. If 2, 2, 14 are added to them respectively, the resulting numbers are G.P. Find the numbers.

③ Find the four numbers in G.P such that the sum of the extremes numbers in 112 and the sum of the middle numbers is 48.

④ If a, b, c d are in G.P ; prove that

i> $a^n + b^n$; $b^n + c^n$; $c^n + d^n$ are in G.P.

ii> $(a^2+b^2+c^2)(b^2+c^2+d^2) = (ab+bc+cd)^2$.

⑤ How many terms of the G.P : $3, \frac{3}{2}, \frac{3}{4} \ldots$ are needed to give the sum $\frac{3069}{512}$.

Home work:— Workout ch- 11B

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