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LEARN TO LEAD

STEPPING STONE SCHOOL (HIGH)

Sub- MATHEMATICS : CLASS – X.

Worksheet – 21. Date – 24.06.20

Topic:– Geometrical Progression (G.P)

Chapter → 11

CONCEPT: When the terms of the sequence are generated from first term by multiplying a fixed number (called common ratio) to the predecessor e.g

2, 4, 8, 16, ------ or more generally if first term is 'a' and common ratio is 'r' then the series:– $a, ar, ar^2, ar^3 ------ ar^{n-1}$ is called Geometrical progression. 'r' may be greater than or less than '1'. E.g the series $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}$ ---- has common ratio $r = \frac{1}{3} < 1$. Hence nth term of G.P, $t_n = ar^{n-1}$.

If a, b, c, d ---- are in G.P then

$\frac{b}{a} = \frac{c}{b} = \frac{d}{c} = r$ = Common ratio

$\Rightarrow b^2 = ac$ or, $c^2 = bd$ i.e. b is the G.M = geometrical mean between 'a' and 'c'.

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Similarly, 'c' is G.M between 'b' and 'd'. The terms in between the first and the last term are called means.

Note: If two numbers 'a' and 'b' s.t $a \neq b$

Arithmetical mean is $\frac{a+b}{2}$ and let GM be x then, $x^2 = ab \Rightarrow x = \pm\sqrt{ab}$. We can show A.M > G.M. Hence, $\frac{a+b}{2} > \sqrt{ab}$

Proof:- As $a \neq b$ $(a-b)^2 > 0 \Rightarrow$

$$(a+b)^2 - 4ab > 0 \Rightarrow (a+b)^2 > 4ab$$

$$\Rightarrow \left(\frac{a+b}{2}\right)^2 > ab \Rightarrow \frac{a+b}{2} > \sqrt{ab}$$ (Taking sq. root of both side)

- If S_n be the sum of nth term of a G.P then

$$S_n = a + ar + ar^2 - - - - - - - - ar^{n-1} \quad \text{(i)}$$

$$r \times S_n = ar + ar^2 + - - - + ar^{n-1} + ar^n \quad \text{(ii)}$$

Subtracting (ii) - (i) we get

$$S_n(r-1) = ar^n - a = a(r^n - 1)$$

$$\Rightarrow S_n = \frac{a(r^n-1)}{r-1} \text{ if } r > 1 \Rightarrow$$

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or $S_n = \frac{a(1-r^n)}{1-r}$ if $r<1$. Eg —

$S_n = 1 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \cdots\cdots = ?$

If add upto nth term of the series it becomes

$$S_n = \frac{1\cdot\left(1-\frac{1}{3^n}\right)}{1-\frac{1}{3}} = \frac{\left(1-\frac{1}{3^n}\right)}{2/3} = \frac{3}{2}\left(1-\frac{1}{3^n}\right)$$

Now if we extend the series upto infinity and as we know $r^n \to 0$ if $r<1$.⇒

So $\frac{1}{3^n} \longrightarrow 0$ if $n \to \infty$. As $r = \frac{1}{3}$

$S_\infty = \frac{3}{2}(1-0) = \frac{3}{2}$. Hence if $r<1$

The sum of the series $= \frac{a(1-0)}{1-r} = \frac{a}{1-r}$ ..

(Remember)

<u>Example</u> 1. Find the seventh term of the G.P

$\sqrt{3}+1,\ 1,\ \frac{\sqrt{3}-1}{2},\ \cdots\cdots$

Ans → Here first term 'a' $= \sqrt{3}+1$ and Common ratio $= r = \frac{1}{\sqrt{3}+1} = \frac{\sqrt{3}-1}{3-1}$ (rationalizing) $=$

$= \frac{\sqrt{3}-1}{2}$ ∴ So applying the formula, $t_n = ar^{n-1}$

$$t_7 = (\sqrt{3}+1)\left(\frac{\sqrt{3}-1}{2}\right)^6 = \frac{(\sqrt{3}+1)(\sqrt{3}-1)}{2^6}\cdot(\sqrt{3}-1)^5 = \frac{1}{2^5}(\sqrt{3}-1)^5$$

$= \frac{1}{32}(\sqrt{3}-1)^5$.

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Example 2: Find the G.P whose 5th term is 48 and 8th term is 384.

Ans → If first term is 'a' and Common ratio be 'r'

then $t_5 = ar^4 = 48$ —— (i)

$t_8 = ar^7 = 384$ —— (ii)

$\Rightarrow \dfrac{ar^7}{ar^4} = \dfrac{384}{48} = 8 \Rightarrow r^3 = 2^3 \Rightarrow r = 2$

So from (i) $a.2^4 = 48 \Rightarrow a = \dfrac{48}{16} = 3$

Hence the series is: 3, 6, 12, ———

Example 3. For which term of G.P;

3, −6, +12, −24 + ... is −384

Ans → Let us assume that nth term is '−384'

So by formula; $t_n = ar^{n-1}$, we get

$-384 = 3.(-2)^{n-1}$ as r = Common ratio $= \dfrac{-6}{3} = -2$

and first term = 3.

$\Rightarrow (-2)^{n-1} = -128$

$\Rightarrow (-2)^{n-1} = (-2)^7 \Rightarrow n-1 = 7 \Rightarrow n = 8.$

Example 4: Find the 4th term from the end of the series

8, 4, 2 ...... $\dfrac{1}{128}$.

Ans : 4 Note: If 'l' be the last term, then a G.P can be written as :–

$a, ar, ar^2 \text{------} \frac{l}{r^2}, \frac{l}{r}, l,$ where 'r' is the common ratio and 'a' is first term. Hence the nth term of the G.P from the last will be –

$l\left(\frac{1}{r}\right)^{n-1} = \frac{l}{r^{n-1}}$ ∴ So in our given sum $l = \frac{1}{128}$ and $r = \frac{4}{8} = \frac{1}{2}$. So by the above formula 4th term from the last $= \frac{l}{r^3} =$

$$= \frac{\frac{1}{128}}{\left(\frac{1}{2}\right)^3} = \frac{1}{128} \times 2^3 = \frac{2^3}{2^7} = \frac{1}{2^4} = \frac{1}{16} \text{ (Ans)}$$

Exercises : ① Find the 10th term of the G.P :–

$12, 4, 1\frac{1}{3}, \text{----}$

② If the third, sixth and the last term of a G.P are 6, 48 and 3072 respectively. Find the number of terms in the G.P.

③ If x, $2x+2$, $3x+3$ are the first three terms of a geometric progression. Find its fourth term.

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(4) The third term of a G.P is '4'. Find the product of its first five terms.

Q5) If the 4th, 10th and 16th terms of G.P are x, y, z respectively. Prove that; x, y, z are in G.P.

Q6) The product of 3rd and 8th term of a G.P is 243. If its 4th term is 3; find its 7th term.

Home work:= Complete Exercise '11 A' from your Text Book. (CONCISE MATHEMATICS).

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