



STEPPING STONE SCHOOL (HIGH)

Standard sums on A.P.

Example : ① If the sum of m th term is $S_m = n$ and the sum of n th term is $S_n = m$ in an A.P., then show that $S_{m+n} = -(m+n)$

Ans \rightarrow Given $S_m = n \Rightarrow$

$$n = \frac{m}{2} [2a + (m-1)d] \Rightarrow 2a + (m-1)d = \frac{2n}{m} \quad \text{---(i)}$$

Similarly, $S_n = m \Rightarrow 2a + (n-1)d = \frac{2m}{n} \quad \text{---(ii)}$

Subtracting, ie, (ii) - (i) \Rightarrow

$$-(m-n)d = \frac{2m}{n} - \frac{2n}{m} = 2 \left(\frac{m^2 - n^2}{mn} \right)$$

$$\Rightarrow -(m-n)d = 2(m+n)(m-n) \div mn$$

$$\Rightarrow d = \frac{-2(m+n)}{mn} \quad \text{---(iii)}$$

Now; $S_{m+n} = \left(\frac{m+n}{2} \right) [2a + (m+n-1)d]$

$$= \left(\frac{m+n}{2} \right) [2a + (m-1)d + nd]$$

$$= \left(\frac{m+n}{2} \right) \left[\frac{2n}{m} - \frac{2(m+n)}{m} \right]; \text{ by (i) and (iii)}$$

$$S_{m+n} = \left(\frac{m+n}{2}\right) \left[\frac{2\{n-m-n\}}{m} \right] \quad \text{--- (P2)}$$

$$= (m+n) \left[-\frac{m}{m} \right] = -(m+n) ; \text{ Proved}$$

Example 2: The p th term of an A.P is 20 and its q th term is 10. Show that the sum of first $(p+q)$ th term is $\frac{p+q}{2} \left\{ 30 + \frac{10}{p-q} \right\}$

Ans → Let the first term and common difference of the A.P are 'a' and 'd'. So; $a + (p-1)d = 20$
and $a + (q-1)d = 10$

Subtracting we get; $(p-q)d = 10$.

$$d = \frac{10}{p-q} \quad \text{--- (i)}$$

Similarly adding we get;

$$2a + (p+q-1)d - d = 30$$

$$\Rightarrow 2a + (p+q-1)d = 30 + d \quad \text{--- (ii)}$$

So the sum of $(p+q)$ th term -

$$S_{p+q} = \left(\frac{p+q}{2}\right) [2a + (p+q-1)d] = \left(\frac{p+q}{2}\right) [30 + d];$$

by (ii)

$$= \left(\frac{p+q}{2}\right) \left[30 + \frac{10}{p-q} \right]; \text{ by (i)}$$

Hence proved.

P.T.O →

Example : 3.

The sum of n , $2n$ and $3n$ terms of an A.P are S_1 , S_2 and S_3 respectively

Prove that ; $S_3 = 3(S_2 - S_1)$

Ans - Let the first term and common difference be 'a' and 'd' respectively. So -

$$S_1 = \frac{n}{2} [2a + (n-1)d] \quad ; \quad S_2 = \frac{2n}{2} [2a + (2n-1)d]$$

$$\text{So } S_2 - S_1 = \frac{n}{2} [4a + (4n-2)d - 2a - (n-1)d]$$

$$3(S_2 - S_1) = \frac{3n}{2} [2a + (3n-1)d]$$

$$= \frac{3n}{2} [2a + (3n-1)d] = S_3, \text{ by definition of } S_n.$$

$$= S_3, \text{ Proved.}$$

Here $n \neq 3$.

Example 4

Find 'a' and 'b' if 12, (a+b), 2a, and 'b' are in A.P

Ans - As 12, (a+b) and 2a are in A.P, So —

$$a+b = \frac{12+2a}{2} \Rightarrow a+b = 6+a \Rightarrow b=6$$

Similarly ; (a+b), 2a, b are in A.P, So —

$$2a = \frac{a+b+b}{2} = \frac{a+2b}{2} \Rightarrow 4a = a+2b \Rightarrow 3a=2b$$

Hence, $3a = 2 \times 6 = 12 \Rightarrow a = 4$.

P.T.O →

(P-4)

Exercises: ① The sum of first 'n' terms of an A.P is $5n^2 - 8n$. Find the A.P and hence find its 15th term.

② The common difference of two different A.P's is '8'. If the difference of their 50th terms is 50. What will be the difference between their 80th terms.

③ In an A.P, the sum of first ten terms is -80 and the sum of next ten terms is -280. Find the A.P.

④ The sum of three ~~terms~~ numbers in A.P is 12 and the sum of their cubes is 216. Find the numbers.

⑤ Show that the sum of an A.P, whose first term is 'a', second term is 'b' and last term is 'c' is equal to $\frac{(a+c)(b+c-2a)}{2(b-a)}$.

⑥ If the sum of first 'm' terms of an A.P is the same as the sum of its first 'n' terms ($m \neq n$), show that; $S_{m+n} = 0$ i.e. sum of its first $(m+n)$ th terms is zero. - END -