



STEPPING STONE SCHOOL (HIGH)

"INSTRUCTION"

Please read the chapter from your textbook and the attached notes. Then work out the exercises neatly in your notebooks henceforth. Make a contents page first with columns under the heads: date, chapter number, chapter name and teacher's signature. Ensure neat and tidy work.

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Date	Worksheet No	Chapter No AND NAME	Page No.	Teacher's Signature

Worksheet - 19 Date - 22.06.20

Topic - A.P.

Example: ① ^{Find} The sum of all natural numbers between 250 and 1000 which are divisible by '9'.

Ans → Note $\frac{250}{9} = 27\frac{7}{9}$. So the first term of the series, greater than 250 and divisible by 9 is $28 \times 9 = 252 = a$ (let)

Similarly: $1000 \div 9 = 111\frac{1}{9}$. So the last term will be '999'. If the number of terms be 'n' then last term $= l = a + (n-1)d$, where 'd' is the common difference $\equiv 9$ here.

$$\text{So } 999 = 252 + (n-1)9 \Rightarrow (n-1)9 = 747 \\ \Rightarrow n-1 = 83 \Rightarrow n = 84.$$

$$\text{So } \text{Sum} = S_n = \frac{n}{2}[a+l] = \frac{84}{2}(252+999) \\ = 42 \times 1251 = 52542 \text{ (Ans.)}$$

Example 2: How many terms of the A.P; $20, 19\frac{1}{3}, 18\frac{2}{3}, \dots$ must be taken so that their sum

Sum is 300. Explain why are there two answers.

Ans → Let the number of terms be 'n'.

$$\text{Given } a = 20, d = 19 \frac{1}{3} - 20 = -\frac{2}{3}$$

$$\text{So } S_n = \frac{n}{2} [2a + (n-1)d] \Rightarrow$$

$$300 = \frac{n}{2} \left[40 - \frac{2}{3}(n-1) \right] = \frac{n}{2} \left[\frac{120 - 2n + 2}{3} \right]$$

$$\Rightarrow 1800 = 122n - 2n^2$$

$$\Rightarrow n^2 - 61n + 900 = 0$$

$$\Rightarrow (n-36)(n-25) = 0 \Rightarrow n = 25 \text{ or } n = 36$$

$$\text{Now, Note } t_{31} = 20 + 30(-\frac{2}{3}) = 0$$

$$t_{26} = 20 + 25(-\frac{2}{3}) = \frac{10}{3} \text{ and}$$

$t_{36} = 20 + 35(-\frac{2}{3}) = -\frac{10}{3}$. So the sum of five terms from 26th to 30th and the sum of five terms from 31st to 36th are same but opposite in sign. Hence the sum from 26th to 36th terms adds 0 to the sum 300 upto 25th term. Hence two answers.

Example 3: The sum of three consecutive terms of an A.P. is 21. The sum of their squares

is 165. Find the A.P.

Ans → Let the terms in A.P. be $a-d$, a , $a+d$

$$\text{Q} \quad (a-d) + a + (a+d) = 21 \Rightarrow 3a = 21 \Rightarrow a = 7$$

$$\text{Now given: } (a-d)^2 + a^2 + (a+d)^2 = 165$$

$$\Rightarrow (a+d)^2 + (a-d)^2 + a^2 = 165$$

$$\Rightarrow 2[a^2 + d^2] + a^2 = 165$$

$$\Rightarrow 3a^2 + 2d^2 = 165 \Rightarrow 3 \times 49 + 2d^2 = 165$$

$$\Rightarrow 2d^2 = 165 - 147 = 18 \Rightarrow 2d^2 = 18$$

$$\Rightarrow d^2 = 9 \Rightarrow d = \pm 3. \text{ Hence the numbers}$$

are either; $4, 7, 10$ or $10, 7, 4$.

Example 4: If $k \neq 0$ and a, b, c are in A.P

i) If $K \neq 0$ and ka, kb, kc are also in A.P. similarly

then ka, kb, kc are also in A.P but the common

$\frac{a}{k}, \frac{b}{k}, \frac{c}{k}$ are also in A.P but the respective

difference will be changed Kd and $\frac{d}{k}$ respectively

if d is common difference of a, b, c

ii) If a, kb, c are in A.P; then $a \neq k, b \neq k$

and $c \neq k$ are also in A.P.

Applying the above two concept prove that \rightarrow

$\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P if (P-4)

$\frac{b+c}{a}, \frac{c+a}{b}, \frac{a+b}{c}$ are in A.P.

Ans → Given $\frac{b+c}{a}, \frac{c+a}{b}, \frac{a+b}{c}$ are in A.P

So $\Rightarrow \frac{b+c}{a} + 1, \frac{c+a}{b} + 1, \frac{a+b}{c} + 1$ are in A.P

$\Rightarrow \frac{a+b+c}{a}, \frac{a+b+c}{b}, \frac{a+b+c}{c}$ are in A.P

$\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P. (Dividing $(a+b+c)$ in each term).

EXERCISES:

① If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P

Show that: bc, ca, ab are also in A.P

② The sum of three numbers in A.P is 15 and the sum of the squares of the extreme terms is 58. Find the numbers.

③ Divide 96 into four parts which are in A.P and the ratio between products of their means to the product of their extremes is $15:7$

[Hints: Assume the terms as: $a-3d, a-d, a+d$ and $a+3d$ and $4a=96 \Rightarrow a=24$ & $- (a^2 - d^2) : (a^2 - 9d^2) = 15:7$]

④

Mrs. Den repays her total loan of Rs. 1,18,000 by paying instalments every month. If the instalment for the first month is Rs. 1000 and it increases by Rs. 100 every month; What amount will she pay in 30th month? What remaining amount will she pay after 30th instalment.

⑤ Find the sum of last 8 terms of the A.P

-12, -10, -8, 58

⑥ For an A.P; Show that:-

$$(m+n)^{\text{th}} \text{ term} + (m-n)^{\text{th}} \text{ term} = 2 \times m^{\text{th}} \text{ term}$$

⑦ The sum of first 14 terms of an A.P is 1050 and its 14th term is 140.

Find the 20th term.

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