ESTD. 2001

LEARN TO LEAD

STEPPING STONE SCHOOL (HIGH)

WORKSHEET – 17 : CLASS – X :

Sub – MATHEMATICS. Date – 11/06/20

Topic:- Arithmetic Progression :

A progression is a sequence of numbers whose each term is related to its predecessor and successor by a uniform law. For example.

Even number series : 2, 4, 6, 8, ------ is an Arithmetical progression (A.P)

Similarly odd number series : 1, 3, 5, 7 ---- is also an A.P of the terms in those series are equispaced with common difference '2'. Similarly common difference (d) in the series : 5, 10, 15, 20 ----- is '5'

So if we denote first term of the series as 'a' and common difference as 'd', so the A.P will be 'a', a+d, a+2d, a+3d -----

Note t_1 = term one = first term = a

t_2 = second term = a+d

t_3 = third " = a+2d ------->

So on nth term $= t_n = a + (n-1)d$ --- (1)
(Remember)

If the last term is t_n $\therefore$ $l = a + (n-1)d$

Then we write the sum of the terms of an A.P. as $S_n = a + (a+d) + (a+2d) + \text{------} (l-d) + l$ --- (i)

or, $S_n = l + (l-d) + (l-2d) + \text{----} (a+d) + a$ --- (ii)

+

Adding (i) + (ii) we get;—

$2S_n = (a+l) + (a+l) + (a+l) + \text{---} + (a+l)$ 'n' times

[Added 'termwise']

$2S_n = n(a+l) \Rightarrow S_n = \frac{n}{2}(a+l)$ —— (2)
(Rember)

So sum of 'n' natural numbers ie

$S_N = 1+2+3+ \text{-----} + n$; goes $S_n = \frac{n}{2}(1+n)$

$S_n = \frac{n(n+1)}{2}$ as $a = 1$ and $l = n$

E.g. $S_{100} = 1+2+ \text{---} 100 = \frac{100(101)}{2} = 5050$

Now again from (2) $S_n = \frac{n}{2}[a + a + (n-1)d]$ if last term $l = a + (n-1)d$ $\therefore$ $S_n = \frac{n}{2}[2a + (n-1)d]$ --- (3)

Remember

If a, b, c are in A.P ; then

$b-a = c-b \Rightarrow 2b = a+c \Rightarrow b = \frac{a+c}{2}$

Here 'b' is called arithmetical mean (A.M) of 'a' & 'c' e.g. A.M between 1, and 9 is $\frac{1+9}{2} = 5$; So the series will be

1, 5, 9, 13 ---- and cd = d = '4'

Examples: ① If n' the term of an A.P is $t_n = 2n-3$, Find the A.P

Ans → $t_n = 2n-3$; so $t_1 = 2-3 = -1$

$t_2 = 2\times2-3 = 1$; $t_3 = 2\times3-3 = 3$

So the series become: $-1, 1, 3, 5$ -----

② If the sum of the terms of an A.P is n^2 : find the series

Ans → $S_n = n^2 \Rightarrow S_1 = 1^2 = 1, = t_1$

$S_2 = t_1 + t_2 = 2^2 = 4 \Rightarrow 1 + t_2 = 4 \Rightarrow$

$t_2 = 3$ --- So the series is : 1, 3, 5, ---

ie 'odd' No. series ; Note applying formula '2' →

ie $S_n = \frac{n}{2}(a+l)$ we get –

$S_n = \frac{n}{2}(1 + 2n-1)$ of odd No. Series has 'n'th term $(2n-1)$.

$S_n = \frac{n}{2}(2n) = n^2$.

As, $t_n = 1 + (n-1)2$
$= 1 + 2n - 2$
$= 2n - 1$

EXERCISES: ① Find the A.P whose second term is '12' and 7th term exceeds the 4th by 15.

② Find the 24th term of the A.P : 12, 10, 8, ------

③ Is 402 a term of the sequence ; 8, 13, 18, 23 ---

④ How many terms are there in the series :–

4, 7, 10, 13 ....

⑤ Which term of an A.P : 1, 4 ; 7 ; 10 ------ is 52

⑥ The 5th and 6th term of an A.P are respectively 6 and 5. Find the 11th term.

⑦ Find the 10th term from the end of the A.P

4, 9, 14 254

→ END ←