ESTD. 2001

LEARN TO LEAD

STEPPING STONE SCHOOL (HIGH)

WORKSHEET – 16. Date - 10.06.20

CLASS – X. Sub – MATHEMATICS

Some special Approach of proportions: —

EXAMPLES ① If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$ then each is equal to $\frac{\sqrt{a^2+c^2+e^2}}{\sqrt{b^2+d^2+f^2}} = \frac{\sqrt[3]{a^3+c^3+e^3}}{\sqrt[3]{b^3+d^3+f^3}} = \cdots$

$$= \frac{\sqrt[n]{a^n+c^n+e^n}}{\sqrt[n]{b^n+d^n+f^n}}.$$

Proof: Let $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k \Rightarrow a = bk$
$c = dk$
$e = fk$

So $a^2+c^2+e^2 = k^2(b^2+d^2+f^2)$

$$\Rightarrow \frac{a^2+c^2+e^2}{b^2+d^2+f^2} = k^2$$

$$\Rightarrow \frac{\sqrt{a^2+c^2+e^2}}{\sqrt{b^2+d^2+f^2}} = k.$$ Similarly other's proof you try.

Application: $\frac{\sin\theta}{\cos\theta+\sin\theta} = \frac{\cos\theta}{\cos\theta-\sin\theta}$ Prove, $\Rightarrow \cos\theta+\sin\theta = \sqrt{2}\sin\theta$

$\cos\theta - \sin\theta = \sqrt{2}\cos\theta$

P.T.O →

Proof: Given relation $\frac{\sin\theta}{\sin\theta+\cos\theta} = \frac{\cos\theta}{\cos\theta-\sin\theta}$; Applying above property we get -

$$\frac{\sin\theta}{\cos\theta+\sin\theta} = \frac{\cos\theta}{\cos\theta-\sin\theta} = \frac{\sqrt{\sin^2\theta+\cos^2\theta}}{\sqrt{(\sin\theta+\cos\theta)^2+(\cos\theta-\sin\theta)^2}}$$

$$= \frac{\sqrt{1}}{\sqrt{2(\sin^2\theta+\cos^2\theta)}} = \frac{1}{\sqrt{2}}$$

So $\frac{\sin\theta}{\cos\theta+\sin\theta} = \frac{\cos\theta}{\cos\theta-\sin\theta} = \frac{1}{\sqrt{2}} \Rightarrow \cos\theta+\sin\theta = \sqrt{2}\sin\theta$

& $\cos\theta-\sin\theta = \sqrt{2}\cos\theta$

Ex-2) If $b^2 = ac$ or a, b, c are in Continued proportion

Prove that; $\frac{a^2+b^2+c^2}{(a+b+c)^2} = \frac{(a-b+c)}{(a+b+c)}$

$$LHS = \frac{a^2+c^2+ac}{(a+b+c)^2} \quad (\because b^2 = ac) = \frac{(a+c)^2-2ac+ac}{(a+b+c)^2}$$

$$= \frac{(a+c)^2-ac}{(a+b+c)^2} = \frac{(a+c)^2-b^2}{(a+b+c)^2} =$$

$$= \frac{(a+c+b)(a+c-b)}{(a+b+c)^2} = \frac{a+c-b}{a+b+c}; \text{ Proved.}$$

Ex-3: Find 'x' if, $16\left(\frac{a-x}{a+x}\right)^3 = \frac{(a+x)}{(a-x)}$

PTO

Given $16 = \left(\frac{a+x}{a-x}\right)^4$

$\Rightarrow \left(\frac{a+x}{a-x}\right)^4 = (\pm 2)^4 \Rightarrow \frac{a+x}{a-x} = \pm 2$

$\Rightarrow \frac{a+x}{a-x} = 2$

$\Rightarrow \frac{2a}{2x} = \frac{2+1}{2-1}$

$\Rightarrow \frac{a}{x} = 3 \Rightarrow x = a/3$

OR

$\frac{a+x}{a-x} = -2$

$\Rightarrow \frac{x+a}{x-a} = 2$

$\frac{2x}{2a} = \frac{2+1}{2-1} = \frac{3}{1}$

$\underline{x = 3a}$

So Ans $\Rightarrow x = a/3,\ 3a$

Ex-4: Given $\frac{x^3+12x}{6x^2+8} = \frac{y^3+27y}{9y^2+27}$; Using properties of proportion prove that, $3x = 2y$

Ans – By Componendo and dividendo we get —

$$\frac{x^3+6x^2+12x+8}{x^3-6x^2+12x-8} = \frac{y^3+9y^2+27y+27}{y^3-9y^2+27y-27}$$

$$\Rightarrow \frac{x^3+3x^2\cdot 2+3x\cdot 2^2+2^3}{x^3-3x^2\cdot 2+3x\cdot 2^2-2^3} = \frac{y^3+3\cdot y^2\cdot 3+3\cdot y\cdot 3^2+3^3}{y^3-3y^2\cdot 3+3y\cdot 3^2-3^3}$$

$$\Rightarrow \frac{(x+2)^3}{(x-2)^3} = \frac{(y+3)^3}{(y-3)^3} \Rightarrow \frac{x+2}{x-2} = \frac{y+3}{y-3}$$

Again applying Componendo &

dividendo we get; $\frac{2x}{2\cdot 2} = \frac{2y}{2\cdot 3} \Rightarrow \frac{x}{2} = \frac{y}{3}$

$\Rightarrow 3x = 2y$

EXERCISES: 1> If $\frac{x^2+y^2}{x^2-y^2} = \frac{17}{8}$ find i> x/y

ii> $\frac{x^3+y^3}{x^3-y^3}$. ② Solve for 'x' from

$$\frac{\sqrt{3x+4}+\sqrt{3x-5}}{\sqrt{3x+4}-\sqrt{3x-5}} = 9$$

Q3> If $\frac{ay-bx}{p} = \frac{cx-az}{q} = \frac{bz-cy}{r}$; prove that; $\Rightarrow \frac{y}{b} = \frac{z}{c} = \frac{x}{a}$

Q4> If $\frac{b+c-a}{y+z-x} = \frac{c+a-b}{z+x-y} = \frac{a+b-c}{x+y-z}$; then prove that each ratio is equal to $\frac{a}{x} = \frac{b}{y} = \frac{c}{z}$

Q5> Solve: $\frac{1+x+x^2}{1-x+x^2} = \frac{62(1+x)}{63(1-x)}$

Q6> If $x = \frac{\sqrt{a+1}+\sqrt{a-1}}{\sqrt{a+1}-\sqrt{a-1}}$; Using properties of proportion

Show that; $x^2-2ax+1=0$.

— END —

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