ESTD. 2001

LEARN TO LEAD

STEPPING STONE SCHOOL (HIGH)

CLASS: IX

SUBJECT - MATH

TOPIC : WORKSHEET # 13

Dated : 29.05.2020

STEPPING STONE SCHOOL (HIGH)

MATHEMATICS

CLASS : 9

WORKSHEET NO: 13 dt: ______

TOPIC : TRIANGLES.

This worksheet is in continuation to WS NO:12, wherein the concept of triangles, types of triangles, terms related to triangles, Congruent triangles, axiom of Congruency and some problems were shown. In this worksheet (NO:13) Some more solved examples of triangles and their properties are given. Please read and study them (theory: NO: 12) and see the Solved examples (WS NO: 13) to build a strong concept of the topic.

TRIANGLES AND PROPERTIES OF TRIANGLES

Ex.1. In ΔABC, $\angle A : \angle B : \angle C = 1 : 2 : 3$. Find the angles and identify the type of the triangle.

Sol. $\angle A : \angle B : \angle C = 1 : 2 : 3$

Let $\angle A$, $\angle B$ and $\angle C$ be x, $2x$ and $3x$ respectively.

We have,

$$\angle A + \angle B + \angle C = 180^\circ \quad \text{[Sum of angles of a } \Delta \text{ is } 180^\circ]$$

$$x + 2x + 3x = 180^\circ$$

$$6x = 180^\circ \Rightarrow x = 30^\circ$$

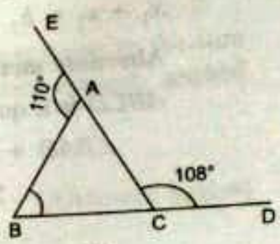
$$\angle A = x = 30^\circ;\ \angle B = 2x = 2 \times 30^\circ = 60^\circ;$$

$$\angle C = 3x = 3 \times 30^\circ = 90^\circ$$

$$\angle A = 30^\circ,\ \angle B = 60^\circ \text{ and } \angle C = 90^\circ$$

Hence, ΔABC is a right-angled triangle at point C.

Ex.2. **ABC is a triangle, in which BC is produced to D, CA is produced to E, $\angle DCA = 108°$ and $\angle BAE = 110°$. Calculate $\angle ABC$.**



Sol. $\angle ACB = 180° - 108°$

$\Rightarrow$ $\angle ACB = 72°$ [Linear pair]

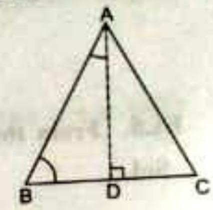
and $\angle BAC = 180° - 110° = 70°$ [Linear pair]

$\angle A + \angle B + \angle C = 180°$

$70° + \angle ABC + 72° = 180°$ [Sum of angles of a Δ is 180°]

$\angle ABC = 180° - 142° = 38°$

Ex.3. **In an equilateral triangle ABC, the bisector of $\angle BAC$ meets BC at D. Find $\angle ADC$.**



Sol. $\angle ABC = 60°$ [An angle of an equilateral Δ]

$\angle BAD = 30°$ [AD is bisector of $\angle BAC$]

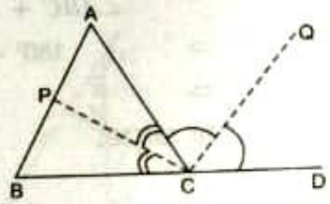
$\therefore$ $\angle ADC = \angle ABD + \angle BAD$

[Exterior angle is equal to the sum of interior opposite angles]

$= 60° + 30° = 90°$

$\Rightarrow$ $\angle ADC = 90°$

Ex.4. **In a ΔABC, BC is produced to D. CP and CQ are bisectors of $\angle ACB$ and $\angle ACD$ respectively. Find $\angle PCQ$.**



Sol. $\angle ACD + \angle ACB = 180°$ [Linear pair]

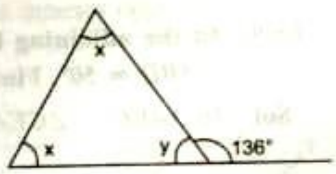
$\frac{1}{2}\angle ACD + \frac{1}{2}\angle ACB = 90°$

$\Rightarrow$ $\angle ACQ + \angle ACP = 90°$

[$\because$ CP and CQ are bisectors of $\angle ACB$ and $\angle ACD$]

$\Rightarrow$ $\angle PCQ = 90°$ [$\because$ $\angle ACQ + \angle ACP = \angle PCQ$]

Ex.5. **From the given figure, find the values of x and y.**



Sol. $y + 136° = 180°$ [Linear pair]

$\Rightarrow$ $y = 180° - 136° = 44°$

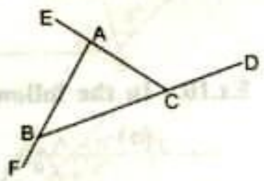
$136° = x + x$

[Exterior angle of a triangle is equal to sum of interior opposite angles]

$\Rightarrow$ $2x = 136° \Rightarrow x = 68°$

$\therefore$ $x = 68°, y = 44°$

Ex.6. **From the given figure, find $\angle ACD + \angle EAB + \angle FBC$.**



Sol. $\angle BAC + \angle EAB + \angle ABC + \angle CBF + \angle ACB + \angle ACD = 540°$

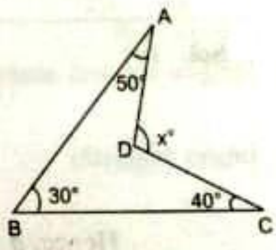
[Three pairs of linear pair]

Now, $\angle EAB + \angle CBF + \angle ACD + (\angle ABC + \angle BAC + \angle ACB) = 540°$

$\angle EAB + \angle CBF + \angle ACD + 180° = 540°$ ($\because$ sum of all interior angles of a Δ is 180°)

$\angle EAB + \angle CBF + \angle ACD = 360°$

Ex.7. **From the given figure, find the value of x.**



Sol. **Construction:** Join BD and produce to E.

Let $\angle ADE = x_1$, $\angle CDE = x_2$

$\angle ABD = b_1$ and $\angle CBD = b_2$

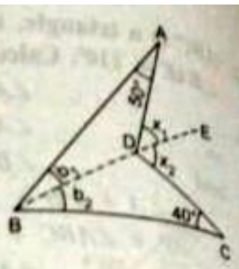
$x_1 = b_1 + 50°$ ...(*i*)

$x_2 = b_2 + 40°$ ...(*ii*)

[Exterior angle of a Δ is equal to sum of interior opposite angles]

Adding (*i*) and (*ii*), we get

$x_1 + x_2 = b_1 + b_2 + 50^\circ + 40^\circ \Rightarrow x = 30^\circ + 90^\circ \Rightarrow x = 120^\circ$



Alternate method:

ABCD is a quadrilateral

$\angle BAD + \angle ADC + \angle DCB + \angle ABC = 360^\circ$

$50^\circ + 360^\circ - x + 40^\circ + 30^\circ = 360^\circ$

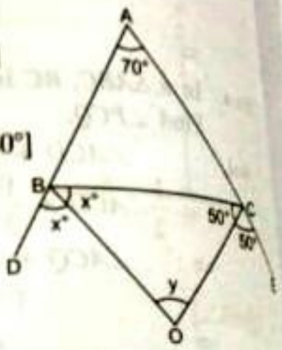
$-x + 480^\circ = 360^\circ$

$-x = 360^\circ - 480^\circ$

$-x = -120^\circ$

$x = 120^\circ$

Ex.8. From the given figure, find *x* and *y*.



Sol. $\angle ABC = 180^\circ - 2x$ [Linear pair]

$\angle ACB = 180^\circ - 100^\circ = 80^\circ$ [Linear pair]

In ΔABC,

$\angle ABC + \angle BAC + \angle ACB = 180^\circ$ [Sum of angles of a Δ is 180°]

$\Rightarrow$ $180^\circ - 2x + 70^\circ + 80^\circ = 180^\circ$

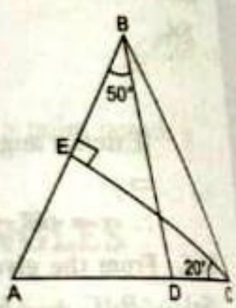
$\Rightarrow$ $-2x = -150^\circ \Rightarrow x = 75^\circ$

In ΔBOC, $x + y + 50^\circ = 180^\circ \Rightarrow 75^\circ + y + 50^\circ = 180^\circ$

$y = 180^\circ - 125^\circ = 55^\circ$

$\Rightarrow$ $x = 75^\circ, y = 55^\circ$

Ex.9. In the adjoining figure, *CE* is perpendicular to *AB*, $\angle ACE = 20^\circ$ and $\angle ABD = 50^\circ$. Find the measure of $\angle BDA$.



Sol. In ΔAEC, $\angle CEA + \angle EAC + \angle ACE = 180^\circ$ [Sum of ∠s of a Δ is 180°]

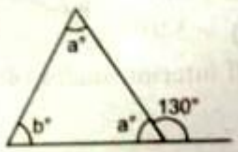
$90^\circ + \angle EAC + 20^\circ = 180 \Rightarrow \angle EAC = 70^\circ$

In ΔABD, $\angle ABD + \angle BDA + \angle BAD = 180^\circ$ [Sum of ∠s of a Δ is 180°]

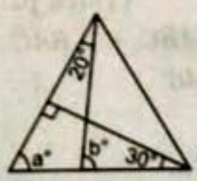
$50^\circ + \angle BDA + 70^\circ = 180 \Rightarrow \angle BDA = 60^\circ$

Ex.10. In the following figures, find the values of *a* and *b*.

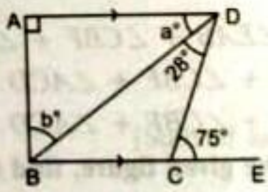
(*a*)



(*b*)



(*c*)



Sol. (*a*) $a + 130^\circ = 180^\circ$ [Linear pair]

$a = 50^\circ$

$a + a + b = 180^\circ$ [Sum of ∠s of a Δ is 180°]

$50^\circ + 50^\circ + b = 180^\circ \Rightarrow b = 80^\circ$

Hence, $a = 50^\circ, b = 80^\circ$

(*b*) $a + 30^\circ + 90^\circ = 180^\circ$ [Sum of ∠s of a Δ is 180°]

$a = 60^\circ$

Also, $b = a + 20^\circ$ [Exterior ∠b is equal to the sum of interior opposite angles]

$b = 60^\circ + 20^\circ = 80^\circ$

Hence, $a = 60^\circ$, $b = 80^\circ$

(*c*) $\angle DBC = \angle ADB = a^\circ$ [Alternate angles]

$a + 28 = 75^\circ$

[Exterior angle of a Δ is equal to the sum of interior opposite angles]

$a = 75^\circ - 28^\circ = 47^\circ$

Also, $\angle B + \angle A = 180^\circ$ [Sum of consecutive interior ∠s is 180°]

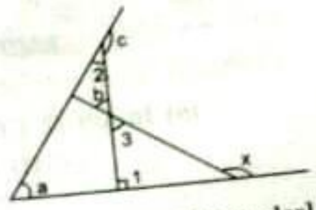
$a + b + 90^\circ = 180^\circ$

$47 + b + 90^\circ = 180^\circ$

$b = 180^\circ - 137^\circ = 43^\circ$

Hence, $a = 47^\circ$, $b = 43^\circ$

Ex.11. From the given figure, find the value of *x* in terms of *a*, *b* and *c*.



Sol. $\angle 2 + c = 180^\circ$ [Linear pair]

$\angle 2 = 180^\circ - c$

Also, $\angle 1 = a + \angle 2$ [Exterior angle is equal to the sum of interior opposite ∠s]

$\angle 1 = a + 180^\circ - c$

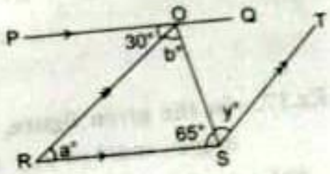
$\angle 3 = b$ [Vertically opposite ∠s]

Also, $x = \angle 1 + \angle 3$ [Exterior angle is equal to the sum of interior opposite angles]

$= a + 180^\circ - c + b$

$x = a + b - c + 180^\circ$

Ex.12. In the given figure, *PQ* || *RS* and *RO* || *ST*. Find *y*.



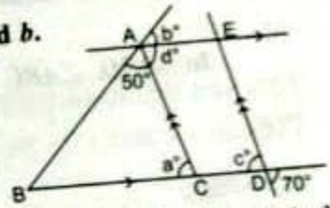
Sol. $a = 30^\circ$ [Alternate interior angles]

Also, $a + b + 65^\circ = 180^\circ$ [Sum of ∠s of a Δ is 180°]

$30^\circ + b + 65^\circ = 180^\circ \Rightarrow b = 85^\circ$

$y = b = 85^\circ$ [Alternate interior angles]

Ex.13. In the given figure, *AE* || *BC* and *CA* || *DE*. Find the values of *a* and *b*.



Sol. Let $\angle CDE = c^\circ$, $\angle CAE = d^\circ$

$c = 70^\circ$ [Vertically opposite ∠s]

$a = c = 70^\circ$ [Corresponding ∠s]

$a = 70^\circ$

$d = a = 70^\circ$ [Alternate interior angles]

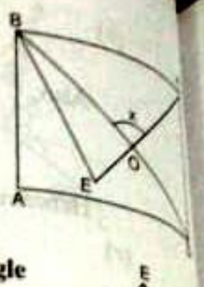
Also, $d = 70^\circ$

Now, $b + d + 50^\circ = 180^\circ$ [Straight angle]

$b + 70^\circ + 50^\circ = 180^\circ \Rightarrow b = 60^\circ$

Hence, $a = 70^\circ$, $b = 60^\circ$

Ex.14. *BEC* is an equilateral triangle in square *ABCD*. Find *x* in degrees.



Sol. $\angle DBC = 45° = \angle OBC$ [*BD* is diagonal and it bisects $\angle ABC$]

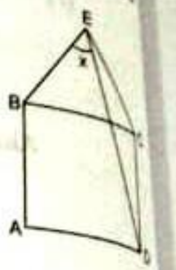
$\angle BCE = 60° = \angle BCO$ [Angle of an equilateral Δ]

In ΔBOC, $\angle CBO + \angle BCO + \angle BOC = 180°$ [Sum of ∠s of a Δ is 180°]

$45° + 60° + \angle BOC = 180°$

$105° + x = 180° \Rightarrow x = 75°$

Ex.15. In the given figure, equilateral ΔEBC surmounts square *ABCD*. Find the angle *BED* represented by *x*.



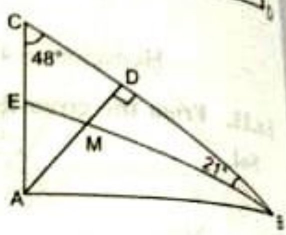
Sol. In ΔECD, $EC = CD$ [Given]

and $\angle ECD = 90° + 60° = 150°$ [$\angle BCE = 60°$, $\angle BCD = 90°$]

So, $\angle CED = \angle EDC = 15°$ [$CE = CD$]

∴ $x = 60° - 15° = 45°$ [$\angle BEC = 60°$]

Ex.16. In the given figure, $\angle A = \angle D = 90°$, $\angle C = 48°$. *BE* is bisector of $\angle B$, *AD* and *BE* intersect at *M*. Calculate: (*a*) $\angle BMD$ (*b*) $\angle AEM$.



Sol. (*a*) In ΔBMD, $\angle BMD + \angle MDB + \angle MBD = 180°$

[Sum of ∠s of a Δ is 180°]

$\angle BMD + 90° + 21° = 180°$

$\angle BMD = 180° - 111° \Rightarrow \angle BMD = 69°$

(*b*) In ΔACD, $\angle CAD + \angle ACD + \angle CDA = 180°$ [Sum of ∠s of a Δ is 180°]

$\angle CAD + 48° + 90° = 180°$

$\angle CAD = 180° - 138° = 42°$

$\angle AME = \angle BMD = 69°$ [Vertically opposite ∠s are equal]

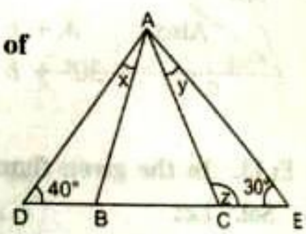
In ΔAME, $\angle EAM + \angle AME + \angle AEM = 180°$ [Sum of ∠s of a Δ is 180°]

$42° + 69° + \angle AEM = 180°$

$\angle AEM + 111° = 180°$

$\angle AEM = 69°$

Ex.17. In the given figure, *ABC* is an equilateral triangle. Find the measures of angles marked *x*, *y* and *z*.



Sol. ΔABC is an equilateral Δ [Given]

$\angle ABC = \angle ACB = \angle BAC = 60°$

In ΔABD, $\angle ABC = 40° + x$ [Exterior angle is equal to the sum of interior opposite angles]

$60° = 40° + x$

$\Rightarrow$ $x = 20°$

In ΔACE, $60° = y + 30°$ [Exterior angle is equal to the sum of interior opposite angles]

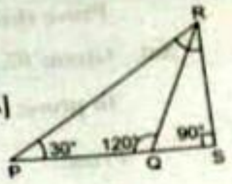
$\Rightarrow$ $y = 30°$

Also, $z + 60° = 180°$ [Linear pair]

$z = 120°$

Hence, $x = 20°$, $y = 30°$, $z = 120°$

ΔPQR, **$\angle P = 30°$, $\angle Q = 120°$ and RS is perpendicular to PQ produced. Show that** **$\angle PRQ = \angle QRS$.**



Given: $RS \perp PQ$ produced, $\angle P = 30°$ and $\angle Q = 120°$

To prove: $\angle PRQ = \angle QRS$

Proof: In ΔPRQ, $\angle RPQ + \angle PRQ + \angle PQR = 180°$ [Sum of $\angle$s of a Δ]

$30° + \angle PRQ + 120° = 180°$

$\angle PRQ = 180° - 150°$

$\angle PRQ = 30°$

In ΔPRS, $\angle PRS + \angle PSR + \angle RPS = 180°$

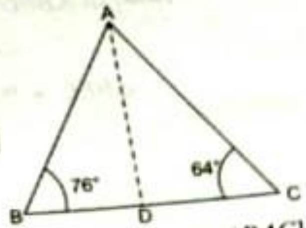
$\angle PRS + 90° + 30° = 180° \Rightarrow \angle PRS = 60°$

$\angle QRS = 60° - 30° = 30°$ [$\angle QRS = \angle PRS - \angle PRQ$]

$\angle PRQ = \angle QRS$

Hence proved.

In ΔABC, $\angle B = 76°$, $\angle C = 64°$ and D is a point on BC, such that AD is bisector of $\angle A$. Find $\angle ADC$.



AD is bisector of $\angle BAC$

In ΔABC, $\angle ABC + \angle ACB + \angle BAC = 180°$ [Sum of $\angle$s of a Δ is 180°]

$76° + 64° + \angle BAC = 180°$

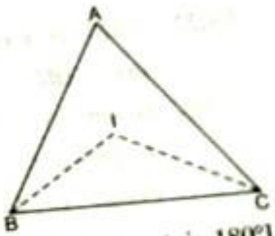
$\angle BAC = 180° - 140° \Rightarrow \angle BAC = 40°$

$\therefore$ $\angle BAD = 20°$ [AD is bisector of $\angle BAC$]

Now $\angle ADC = \angle ABD + \angle BAD = 76° + 20° = 96°$

$\angle ADC = 96°$

In the given figure, the bisectors BI and CI of the angle B and C of ΔABC meet in I. Prove that $\angle BIC = 90° + \frac{\angle A}{2}$.



Given: BI and CI are bisectors of $\angle ABC$ and $\angle ACB$.

To prove: $\angle BIC = 90° + \frac{\angle A}{2}$

Proof: In ΔABC, $\angle ABC + \angle ACB + \angle BAC = 180°$ [Sum of $\angle$s of a Δ is 180°]

$\frac{1}{2}\angle ABC + \frac{1}{2}\angle ACB + \frac{1}{2}\angle BAC = 90°$

$\frac{1}{2}\angle ABC + \frac{1}{2}\angle ACB = 90° - \frac{1}{2}\angle BAC$ ...(*i*)

$\angle IBC + \angle ICB = 90° - \frac{1}{2}\angle BAC$

[BI and CI are bisectors of $\angle B$ and $\angle C$]

In ΔBIC, $\angle BIC + \angle IBC + \angle ICB = 180°$ [Sum of $\angle$s of a Δ is 180°]

$\angle BIC + 90° - \frac{1}{2}\angle BAC = 180°$ [From (*i*)]

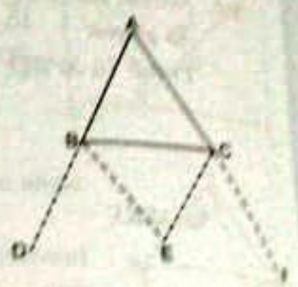
$\angle BIC = 180° - 90° + \frac{1}{2}\angle BAC$

$\angle BIC = 90° + \frac{1}{2}\angle BAC$

$\angle BIC = 90° + \frac{1}{2}\angle A$

Hence proved.

Ex.21. **In ΔABC, AB and AC are produced to D and F. The bisectors of angles BCF and CBD meet at E. Prove that $\angle BEC = 90° - \frac{\angle A}{2}$.**



Sol. **Given:** BE and CE are bisectors of $\angle DBC$ and $\angle BCF$.

To prove: $\angle BEC = 90° - \frac{1}{2}\angle A$

Proof:

$\angle DBC = 180° - \angle B$ [Linear pair]

$\angle BCF = 180° - \angle C$ [Linear pair]

$\angle CBE = \frac{1}{2}\angle DBC = \frac{1}{2}(180° - \angle B) = 90° - \frac{1}{2}\angle B$

$\angle BCE = \frac{1}{2}\angle BCF = \frac{1}{2}(180° - \angle C) = 90° - \frac{1}{2}\angle C$

Now, in ΔBEC [Sum of $\angle$s of a Δ]

$$\angle BEC + \angle ECB + \angle CBE = 180°$$

$$\angle BEC + 90° - \frac{1}{2}\angle C + 90° - \frac{1}{2}\angle B = 180°$$

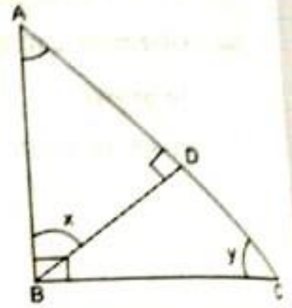
$$\angle BEC = \frac{1}{2}\angle B + \frac{1}{2}\angle C = \frac{1}{2}(\angle B + \angle C)$$

$$= \frac{1}{2}(180° - \angle A) \quad [\angle B + \angle C = 180° - \angle A]$$

$$\angle BEC = 90° - \frac{1}{2}\angle A$$

Hence proved.

Ex.22. **ΔABC is a right-angled triangle at B, and BD is drawn perpendicular to AC. Prove that**
(a) $\angle ABD = \angle C$ **(b) $\angle CBD = \angle A$.**



Sol. **Given:** In right-angled ΔABC, $BD \perp AC$ and $\angle B = 90°$

To prove: (a) $\angle ABD = \angle C$, (b) $\angle CBD = \angle A$

Proof: (a) Let $\angle ABD = x°$, $\angle ACB = y°$

$\angle DBC = 90° - x°$ [$\angle ABC = 90°$]

So, $90° - x° + y° = 90°$ [$\angle DBC + \angle BCD = 90°$]

$x° = y°$

Hence, $\angle ABD = \angle BCD$

(b) We have shown that, $x° = y°$

In ΔDBC, $\angle DBC = 90° - x°$

In ΔBAD, $\angle BAD = 90° - y°$

$x° = y°$

Hence, $\angle DBC = \angle A$

Practice Questions

1. PQR is a triangle in which QR is produced to S, RP is produced to T, $\angle PRS = 105°$ at $\angle QPT = 112°$. Calculate $\angle PQR$.
2. In ΔABC, $\angle A : \angle B : \angle C = 2 : 3 : 4$. Find the angles and identify the type of the triangle.

3. In ΔABC, BC is produced to D. CE and CF are respectively bisectors of $\angle ACB$ and $\angle ACD$. If $\angle ACB = 60^\circ$, find $\angle ECF$.

4. In the given fig. (*i*), find the values of x and y.

5. From the given fig. (*ii*), find the value of y in term of x.

6. From the given fig. (*iii*), find the values of x, y and z.

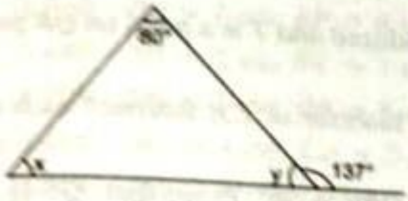


Fig. (*i*)

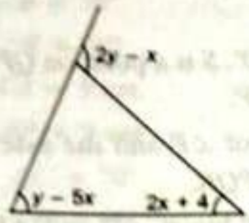


Fig. (*ii*)

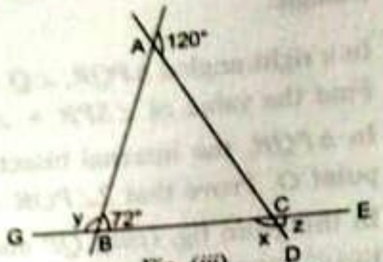


Fig. (*iii*)

7. From the given fig. (*iv*), find x and y.

8. In the given fig. (*v*), PQR is a triangle in which PS is the bisector of $\angle QPR$ and PT is perpendicular to QR. Find $\angle SPT$.

9. In the given fig. (*vi*) ΔABC, OB and OC bisect the exterior angles at B and C respectively. Find $\angle BOC$.

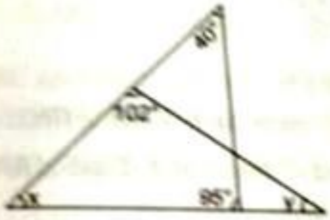


Fig. (*iv*)

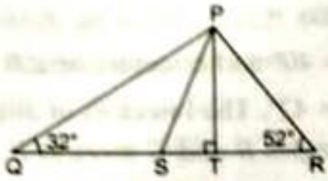


Fig. (*v*)

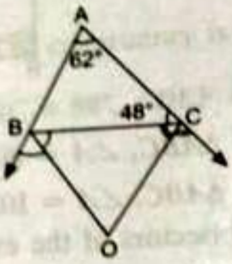
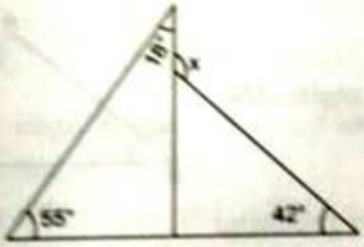


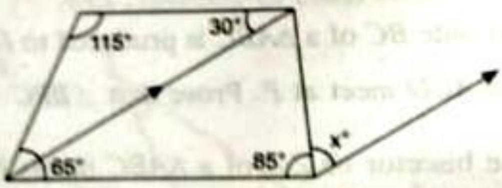
Fig. (*vi*)

10. In the following figures, find x:

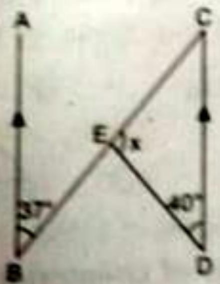
(*a*)



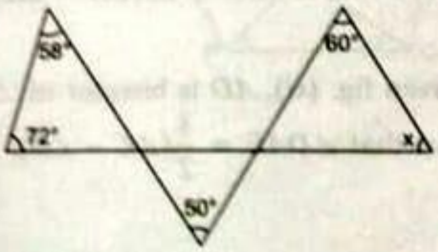
(*b*)



(*c*)



(*d*)



11. In ΔABC, $\angle A = 30^\circ$, $\angle C = 20^\circ$ and CD is perpendicular to AB produced. Find $\angle BCD$.