STEPPING STONE SCHOOL (HIGH)

ESTD. 2001

LEARN TO LEAD

CLASS: IX

SUBJECT - MATH

TOPIC : WORKSHEET # 12

Dated : 21.05.2020

STEPPING STONE SCHOOL (HIGH)

MATHEMATICS

CLASS - 9.

WORKSHEET NO. 12 dt: 28/05/20.

TOPIC : TRIANGLES.

THEORY.

Triangle: A triangle is a plane figure bounded by three line segments. These three lines are called sides of the triangle and the points where they meet are called vertices of the triangle.

In the given figure, ABC is a triangle. In which AB, BC and CA are the three sides. *A*, *B* and *C* are called three vertices.

$\angle BAC$, $\angle ABC$ and $\angle ACB$ are the three angles of the triangle.

Intersecting lines: If two lines meet at a common point in a plane, then they are called intersecting lines and point where the lines meet is called the point of intersection.

Concurrent lines: Three or more lines passes through the same point in a plane are called concurrent lines. The common point is called the point of concurrence of the lines.

C.P.C.T.: It refers to 'corresponding part of congruent triangles are congruent'.

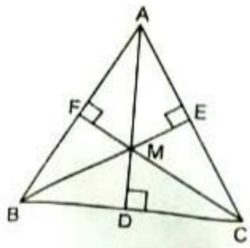
Postulate: Postulate means the assumption specific to geometry and is a statement that is accepted as true that forms the base of a theory, i.e. proof of statement is required.

Axiom: Axiom means the common notations refers to the magnitude of some kind. It is a rule or principle that most people believe to be true, i.e. proof for such rules or principles are not required.

1. **Types of Triangles on the Basis of Sides:**
   - *(a)* **Scalene triangle:** A triangle with all sides unequal, is called scalene triangle.
   - *(b)* **Isosceles triangle:** A triangle having two sides equal, is called an isosceles triangle.
   - *(c)* **Equilateral triangle:** If all the three sides of a triangle are equal, then it is called an equilateral triangle.
2. **Types of Triangles on the Basis of Angles:**
   - *(a)* **Acute-angled triangle:** If all the three angles of a triangle are less than 90°, then it is called acute-angled triangle.
   - *(b)* **Right-angled triangle:** If one angle of a triangle is equal to 90°, then it is called right-angled triangle.
   - *(c)* **Obtuse-angled triangle:** If one angle of a triangle is obtuse (greater than 90°), then it is called an obtuse-angled triangle.
3. **Altitude:** Perpendicular drawn from a vertex of a triangle to the opposite side, is called an altitude of the triangle.

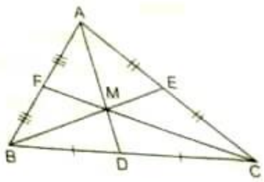
   A triangle *ABC* has maximum three altitudes, i.e. *AD*, *BE* and *CF*.

   All three altitudes of a triangle passes through the same point. This point is called the Orthocentre of the triangle. In adjoining figure, *M* is the orthocentre of ΔABC.



4. **Median:** In a triangle the straight line which join a vertex and the mid-point of the opposite side is called the median.

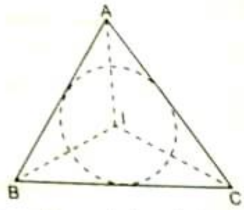
There are 3 medians in a triangle. In adjoining figure, AD, BE and CF are the medians of ΔABC.



All the three medians passes through the same point. This point of intersection is called the centroid of the triangle. Here M is the centroid of ΔABC.

$AM : MD = CM : MF = BM : ME = 2 : 1$

5. **Incentre and Incircle:** Line bisecting an interior angle of a triangle is called bisector (internal) of the angle of the triangle. AI is the internal bisector of the $\angle A$. $\angle BAI = \angle IAC$

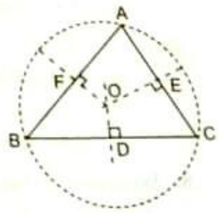


A triangle has three internal bisectors of its interior angles.

All the three bisectors of the triangle passes through the same point. This point of intersection is known as incentre of the triangle.

Incentre is the centre of the circle, which touches all the sides of ΔABC internally and the circle is called Incircle.

6. **Circumcentre and Circumcircle:** Line bisecting a side of a triangle and perpendicular to it is called the right bisector.



The point of intersection of three right bisectors of a triangle is called Circumcentre and the circle drawn with 'O' as centre touching all the vertices of the triangle is called its circumcircle. The circumcircle touches all the three vertices of the triangle.

Theorems Statement:

1. The sum of three angles of a triangle is 180°.
2. If a side of a triangle is produced, the exterior angle so formed is equal to the sum of the two interior opposite angles.

Points to Remember:

(*a*) A triangle cannot have more than one right angle.

(*b*) An exterior angle of a triangle is greater than either of the two interior opposite angles.

(*c*) A triangle cannot have more than one obtuse angle.

(*d*) Every triangle has at least two acute angles.

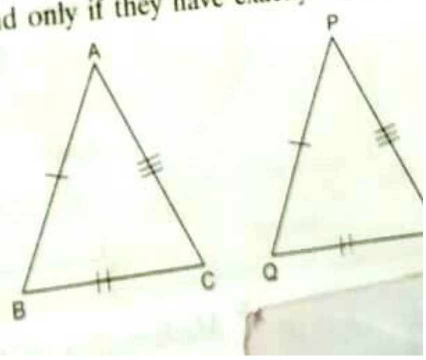
(*e*) In a right-angled triangle, the sum of two acute angles is 90°.

(*f*) In an equilateral triangle, bisector of any of the angle is always perpendicular to opposite base side.

(*g*) In an isosceles triangle, bisector of vertical angle is always perpendicular to the base.

(*h*) If two angles of a triangle are equal to two angles of another triangle, then the third angle of both the triangles will also be equal.

7. **Congruent Triangles:** Two triangles are called congruent if and only if they have exactly the same shape and size, e.g. in given ΔABC and ΔPQR,



if $AB = PQ;\ BC = QR;\ AC = PR$

and $\angle A = \angle P,\ \angle B = \angle Q,\ \angle C = \angle R$

Then, $\Delta ABC \cong \Delta PQR$.

The symbol $\cong$ (read as " congruent to") is used to indicate

Note:

1. Congruent triangles are **"equal in all respects".**
2. If $\Delta ABC \cong \Delta PQR$, then any one triangle cover the other exactly.
3. In congruent triangles corresponding sides and corresponding angles are equal.
4. The corresponding sides lie opposite to the equal angles and the corresponding angles lie opposite to equal sides.

Conditions for Congruency of Triangles:

(a) **SAS (Side-Angle-Side) axiom of congruency:** Two triangles are congruent if two sides and the included angle of one triangle are equal to two sides and included angle of the other triangle.

(b) **ASA (Angle-Side-Angle) axiom of congruency:** Two triangles are congruent if two angles and the included side of one triangle are equal to the two angles and the included side of the other triangle.

(c) **AAS (Angle-Angle-Side) axiom of congruency:** Two triangles are congruent if any two angles and a (non–included) side of one triangle are equal to two angles and corresponding side of the other triangle. (The equality of corresponding sides is essential)

(d) **SSS (Side-Side-Side) axiom of congruency:** Two triangles are congruent if all the three sides of one triangle are equal to the corresponding three sides of the other triangle.

(e) **RHS (Right Angle-Hypotenuse-Side) axiom of congruency:** Two right-angled triangles are congruent if the hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle.

8. Isosceles Triangle: A triangle in which two sides are equal is called an isosceles triangle. Angles opposite to equal sides are equal. These angles are called base angles.

Theorems Statement:

1. If two sides of a triangle are equal, then the angles opposite to them are also equal.
2. In an equilateral triangle, each angle is 60°.
3. If two angles of a triangle are equal, then the sides opposite to them are also equal.

Inequality Properties of a Triangle:

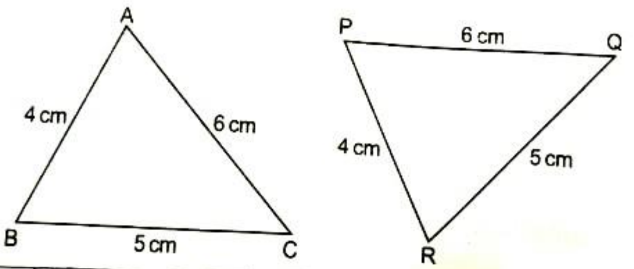
Theorems Statement:

1. If two sides of a triangle are unequal, the greater side has greater angle opposite to it.
2. If two angles of a triangle are unequal, the greater angle has greater side opposite to it.
3. The sum of any two sides of a triangle is greater than the third side.
4. Of all the straight lines that can be drawn to a given straight line from a point outside it, the perpendicular is the shortest.

Axiom of Congruency:

llustration through cut outs:

1. Side-Side-Side (SSS) Axiom



In ΔABC, $AB = 4$ cm, $BC = 5$ cm, $AC = 6$ cm

In ΔPQR, $PQ = 6$ cm, $PR = 4$ cm, $QR = 5$ cm

If we cut ΔPRQ and superimpose it over ΔABC as shown in fig. (*a*).

ΔABC is hidden by ΔPRQ.

Hence, $\Delta ABC \cong \Delta PRQ$ (S.S.S.)

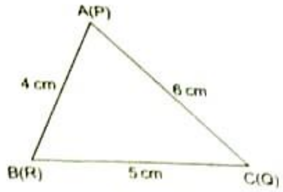
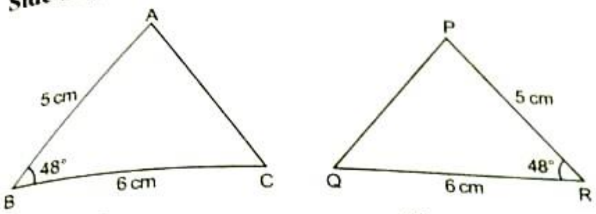


Fig. (a)

2. Side-Angle-Side (SAS)



In ΔABC, $AB = 5$ cm, $BC = 6$ cm, $\angle ABC = 48°$

In ΔPQR, $QR = 6$ cm, $PR = 5$ cm, $\angle PRQ = 48°$

If we cut ΔPRQ and superimpose it over ΔABC as shown in fig. (*b*).

We see that,

$\Delta ABC \cong \Delta PRQ$ (SAS)

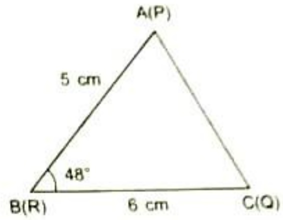
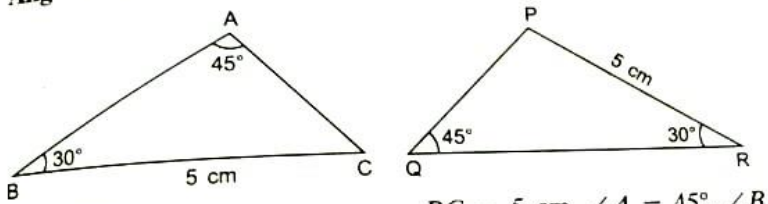


Fig. (b)

3. Angle-Angle-Side (AAS)



In ΔABC, $BC = 5$ cm, $\angle A = 45°$, $\angle B = 30°$

In ΔPQR, $PR = 5$ cm, $\angle Q = 45°$, $\angle R = 30°$

If we cut ΔQRP and superimpose it over ΔABC as shown in fig. (*c*).

We see that, $\Delta ABC \cong \Delta QRP$ (*AAS*)

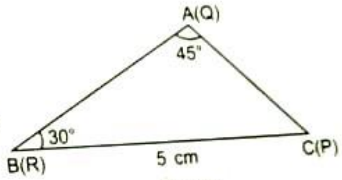
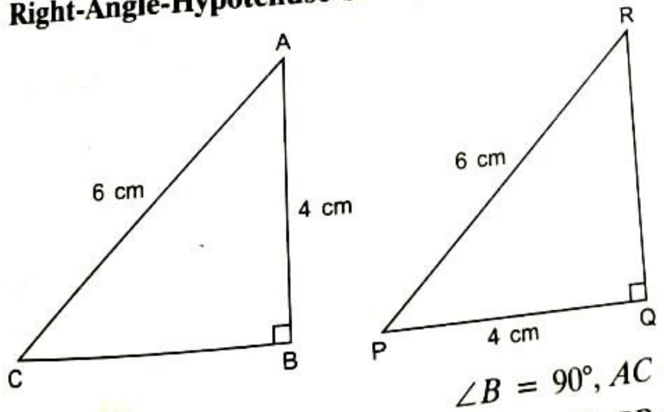


Fig. (c)

4. Right-Angle-Hypotenuse-Side (RHS)



In ΔABC, $\angle B = 90°$, $AC = 6$ cm, $AB = 4$ cm

In ΔPQR, $\angle Q = 90°$, $PR = 6$ cm, $PQ = 4$ cm

If we cut ΔPQR and superimpose it over ΔABC as shown in fig. (*d*).

We see that, $\Delta ABC \cong \Delta PQR$ (RHS)

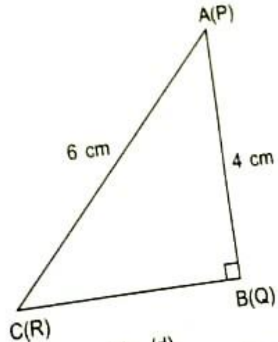
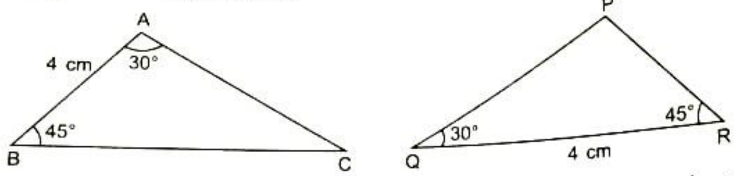


Fig. (d)

5. **Angle-Side-Angle (ASA)**



In ΔABC, $\angle A = 30°, \angle B = 45°\ AB = 4$ cm

In ΔPQR, $\angle Q = 30°, \angle R = 45°, QR = 4$ cm

If we cut ΔPQR and superimpose it over ΔABC as shown in fig. (*e*).

We see that, $\Delta ABC \cong \Delta QRP$ (ASA)

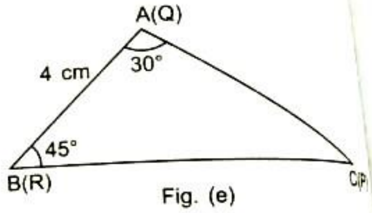
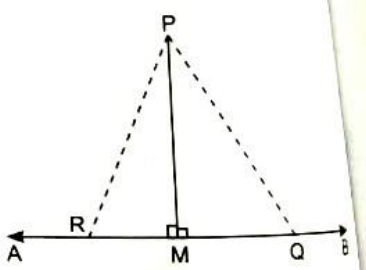


Fig. (e)

6. Of all straight lines that can be drawn to a given line from a point outside it, the perpendicular is the shortest.



Given: A line $\overleftrightarrow{AB}$ and P is a point outside it.

Construction: Draw Perpendicular PM on AB

Join P to Q

Join P to R

To prove: PM is the shortest of all line segments that can be drawn from P on $\overleftrightarrow{AB}$.

Proof: In ΔPMQ,

$$\angle PMQ > \angle PQM \qquad [\angle PMQ = 90°]$$

$$PQ > PM$$

In ΔPMR,

$$\angle PMR > \angle PRM \qquad [\angle PMR = 90°$$

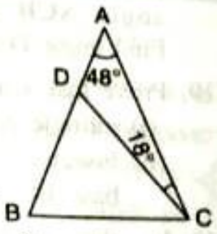
$$PR > PM$$

Hence, PM is the shortest line segment that can be drawn to a line from a point outside it.

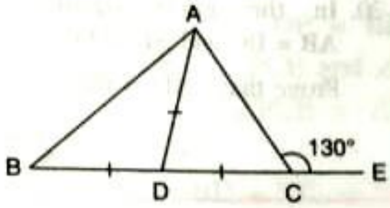
PRACTICE PROBLEMS.

TIME : 60 min

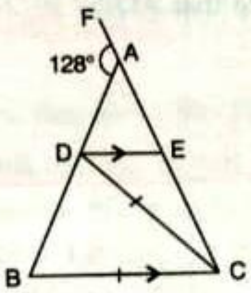
1. In the figure alongside,
   AB = AC
   ∠A = 48° and
   ∠ACD = 18°.
   Show that : BC = CD.



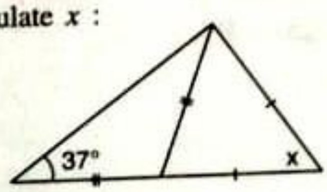
2. Calculate :
   (i) ∠ADC
   (ii) ∠ABC
   (iii) ∠BAC



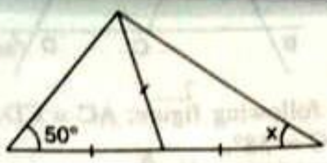
3. In the following figure, AB = AC; BC = CD and DE is parallel to BC. Calculate :
   (i) ∠CDE
   (ii) ∠DCE



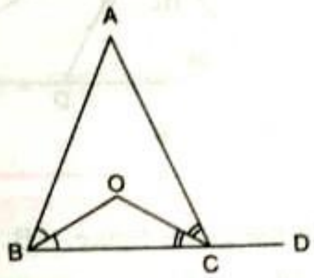
4. Calculate x :
   (i)



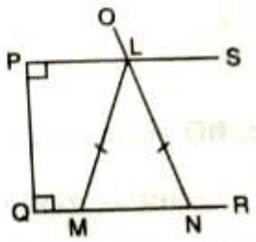
(ii)



5. In the figure, given below, AB = AC.
   Prove that : ∠BOC = ∠ACD.



6. In the figure given below, LM = LN; angle PLN = 110°. Calculate :
   (i) ∠LMN
   (ii) ∠MLN



7. An isosceles triangle ABC has AC = BC. CD bisects AB at D and ∠CAB = 55°.
   Find : (i) ∠DCB (ii) ∠CBD.